



Norwegian University of
Science and Technology

TWO-ROUND THRESHOLD LATTICE SIGNATURES FROM THRESHOLD HOMOMORPHIC ENCRYPTION

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Contents

Threshold Signatures

t-out-of- n **Challenges**

CPA *t-out-of- n* Encryption

Passive Signature Scheme

Active Signature Scheme

Performance

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Threshold Cryptography

The goal is that secrets are shared between n parties, and that any threshold $1 \leq t \leq n$ can jointly compute a decryption or signature based on their shares.

This gives security against an adversary corrupting at most $t - 1$ parties which cannot complete the computation on its own, and robustness if at least t honest parties are available for the computation to be completed.

Applications

On behalf of a set of people/devices/organizations a threshold can...

- ▶ sign transactions and legal documents
- ▶ sign authentication challenges or certificates
- ▶ decrypt ballots in an electronic voting system
- ▶ run pre-processing phases for MPC protocols

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The private key is a short $\mathbf{s} \in R_q^{\ell+k}$, and the verification key consists of a matrix $\bar{\mathbf{A}} := [\mathbf{A} \mid \mathbf{I}] \in R_q^{k \times (\ell+k)}$ and vector $\mathbf{y} := \bar{\mathbf{A}}\mathbf{s}$. The protocol proceeds as follows:

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4. The verifier accepts iff \mathbf{z} is short and $\bar{\mathbf{A}}\mathbf{z} = c \cdot \mathbf{y} + \mathbf{w}$.
5. \rightarrow Non-interactive signature scheme if $c = H(\text{pk}, \mathbf{w}, m)$.

Basic n -out-of- n Threshold Scheme

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3. Each signer then computes $z := \sum_{i \in [n]} z_i$ and outputs the signature (c, z) .

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- ▶ Individual secrets must be short to allow rejection sampling
- ▶ The sum of short elements is also short, but...
- ▶ Secret shared elements are uniformly random

Issues with Random Oracles

Fiat-Shamir signatures require a random oracle to produce challenges, and we cannot evaluate a random oracle using MPC, ZKP, or FHE in a black-box way.

We need a homomorphism to share and combine secrets, but we want to evaluate the random oracle on public input (using communication).

Issues with Zero-Knowledge

Signatures are only (honest-verifier) zero-knowledge when no parties abort.

Then the commit message cannot be sent in the clear if anyone aborts.

But we only learn if anyone aborts after we have computed the challenge...

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- ▶ $KGen_{BGV}$: Sample a uniform element $a \in R_q$ along with $s, e \leftarrow D_{KGen}$, and output the public key $pk := (a, b) = (a, as + pe)$ and secret key $sk := s$.

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- ▶ Enc_{BGV} : On input a public key $\text{pk} = (a, b)$ and a message $m \in R_p$, sample $r, e', e'' \leftarrow D_{\text{Enc}}$ and output the ciphertext $(u, v) = (ar + pe', br + pe'' + m)$.

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- ▶ Dec_{BGV} : On input a secret key $sk = s$ and a ciphertext (u, v) , output the message $m := (v - su \bmod q) \bmod p$.

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3. \mathcal{P}_i computes $b := \sum b_j$, $s'_i = \sum s_{j,i}$, and outputs $\text{pk} = (a, b)$ and $\text{sk}_i = s'_i$.

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TDec On input a ciphertext $\text{ctx} = (u, v)$, a decryption key share $\text{sk}_i = s_i$, and a set of users \mathcal{U} of size t , compute $m_i := \lambda_i s_i u$ using Lagrange coefficient λ_i .

Sample noise $E_i \leftarrow R_q$ s.t. $\|E_i\|_\infty \leq 2^{\text{sec}} B_{\text{Dec}}$, then output $d_i := m_i + pE_i$.

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Comb On input a ciphertext $\text{ctx} = (u, v)$ and a set of partial decryption shares $\{d_j\}_{j \in \mathcal{U}}$, it outputs $m := (v - \sum_{j \in \mathcal{U}} d_j) \bmod p$.

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- ▶ Use linearly homomorphic encryption to combine shares
- ▶ Use t -out-of- n threshold decryption to reconstruct signatures

Main Idea

Keys s and $(\bar{\mathbf{A}}, \mathbf{y} := \bar{\mathbf{A}}s)$ are as before. Instead of sharing s , signers will hold an encryption $\text{ctx}_s = \text{Enc}(s)$ and share the decryption key \mathbf{k} in a t -out-of- n fashion:

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2. Each signer computes $\mathbf{w} := \sum_{i \in \mathcal{U}} \mathbf{w}_i$, $c = H(\mathbf{w})$, and “encrypted signature” $\text{ctx}_{\mathbf{z}} := c \cdot \text{ctx}_s + \sum_{i \in \mathcal{U}} \text{ctx}_{\mathbf{r}_i}$. It sends its threshold decryption share of $\text{ctx}_{\mathbf{z}}$.

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3. Given decryption shares from all parties, each signer can decrypt ctx_z to obtain z , and output the signature (c, z) .

Passive Protocol

Sign_{TS}(sk_i, aux, U, μ)

$r_{i,1}, r_{i,2} \leftarrow D_r, \quad \mathbf{r}_i := [r_{i,1} \ r_{i,2}]$

$w_i := \langle \mathbf{a}, \mathbf{r}_i \rangle, \quad \text{ctx}_{\mathbf{r}_i} := \text{Enc}(\text{pk}_{\mathcal{E}}, \mathbf{r}_i) \xrightarrow{w_i, \text{ctx}_{\mathbf{r}_i}}$

$w := \sum_{j \in \mathcal{U}} w_j, \quad c := H(w, \text{pk}, \mu) \xleftarrow{\{(w_j, \text{ctx}_{\mathbf{r}_j})\}_{j \in \mathcal{U} \setminus \{i\}}}$

$\text{ctx}_{\mathbf{z}} := c \cdot \text{ctx}_{\mathbf{s}} + \sum_{j \in \mathcal{U}} \text{ctx}_{\mathbf{r}_j}$

$\text{ds}_i := \text{TDec}(\text{ctx}_{\mathbf{z}}, \text{sk}_i, \mathcal{U}) \xrightarrow{\text{ds}_i}$

$\mathbf{z} := \text{Comb}(\text{ctx}_{\mathbf{z}}, \{\text{ds}_j\}_{j \in \mathcal{U}}) \xleftarrow{\{\text{ds}_j\}_{j \in \mathcal{U} \setminus \{i\}}}$

return $\sigma := (c, \mathbf{z})$

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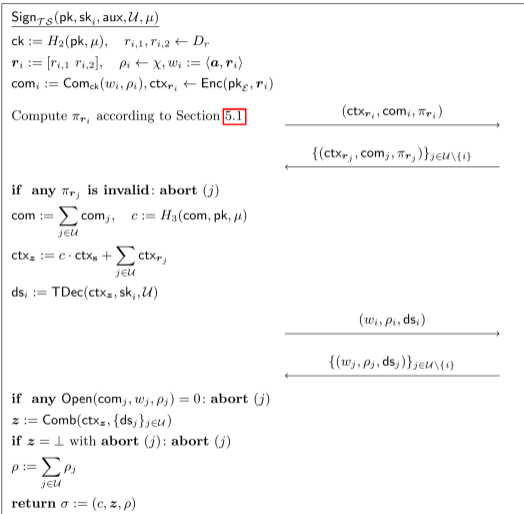
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- ▶ Use zero-knowledge proof to ensure correct computation
- ▶ Use straight-line extractable ZKPs for parallel execution

Actively Secure Signing Protocol



Contents

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Performance

Setting

- ▶ Signing threshold of $t = 3$ out of $n = 5$ signers
- ▶ Signing at most 1 or 365 or 2^{64} signatures total
- ▶ Comparing to Dilithium: n keys and t signatures
- ▶ Focus on signature and key size, not communication

Performance Estimates

Comm.	σ_1	y_1	Π_1	σ_β	y_β	Π_β
Size	4 KB	3 KB	≈ 750 KB	9 KB	7.5 KB	≈ 750 KB
Comm.	σ_∞	y_∞	Π_∞	σ_{triv}	y_{triv}	Π_{triv}
Size	20 KB	14 KB	≈ 1.5 MB	7.3 KB	6.6 KB	2.4 KB

We present sizes for 3-out-of-5 threshold signatures, where $\beta = 365$ times.

We assume a trusted setup, only allow for sequential execution, and give a rough estimate for communication sizes. An optimistic approach reduces communication by 50 % to the potential cost of 3 rounds of interaction.

Future Work

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- ▶ Instantiate the distributed key generation protocol as well
- ▶ Detail the communication and optimize parameters and proofs
- ▶ Make sure all proofs are online extractable for parallel composition
- ▶ Implementing the scheme for more thresholds and signature bounds

Thank you! Questions?

The paper is available at: <https://eprint.iacr.org/2023/1318>