NTNU | Norwegian University of Science and Technology

OLINGO: THRESHOLD LATTICE SIGNATURES WITH DKG AND IDENTIFIABLE ABORT

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Caroline Sandsbråten and **Tjerand Silde**

Papers

This presentation is based on two works:

- *Two-Round Threshold Lattice-Based Signatures from Threshold Homomorphic Encryption*, published at PQCrypto 2024, with Gur and Katz
- *Olingo: Threshold Lattice Signatures with DKG and Identifiable Abort*, soon to appear at IACR ePrint, with Gur, Hough, Katz and Sandsbråten

Contents

Threshold Cryptography Setting

The goal is that secrets are shared between n parties, and that any threshold $1 \leq t \leq n$ can jointly compute a decryption or signature based on their shares.

This gives security against an adversary corrupting at most $t - 1$ parties which cannot complete the computation on its own, and robustness if at least t honest parties are available for the computation to be completed.

Threshold Cryptography Applications

Some potential practical applications are:

- sign transactions and legal documents
- sign authentication challenges or certificates
- decrypt ballots in an electronic voting system
- run pre-processing phases for MPC protocols

Contents

Self-Target MSIS

Definition 1 (SelfTargetMSIS [DKL+18]). *Let k, ℓ be positive integers and $0 < \eta \ll q$. Let $\mathsf{H}\colon R_q^k \times \{0,1\}^* \to \mathcal{C}_\nu$ be a cryptographically secure hash function modeled as a random oracle. Then, given $(\mathbf{A}, \mathbf{t}) \in R_q^{k \times \ell} \times R_q^k$, the Self-Target MSIS problem asks an adversary $\mathcal{A}$ to find $(\mathbf{z}, \mu) \in R_q^\ell \times \{0,1\}^*$ such that $0 < \|\mathbf{z}\|_2 \leq \eta$ and $\mathsf{H}([\mathbf{A} \mid \mathbf{t}] \cdot \mathbf{r}, \mu) = c$, where $\mathbf{r} = [\mathbf{z}\ c]^\top$. $\mathcal{A}$ is said to have advantage $\epsilon_{\mathsf{STMSIS}}$ in solving $\mathsf{SelfTargetMSIS}_{k,\ell,\eta}$ if*

$$\Pr\left[\begin{array}{c}\mathsf{H}([\mathbf{A} \mid \mathbf{t}] \cdot \mathbf{r}, \mu) = c \\ \wedge\ 0 < \|\mathbf{z}\|_2 \leq \eta\end{array} \;\middle|\; \mathbf{A} \leftarrow R_q^{k \times \ell};\ \left(\mathbf{r} = \begin{bmatrix}\mathbf{z} \\ c\end{bmatrix}, \mu\right) \leftarrow \mathcal{A}^{\mathsf{H}}(\mathbf{A}, \mathbf{t})\right] \geq \epsilon_{\mathsf{STMSIS}}.$$

MLWE with Hints

Definition 2 (H-MLWE [KLSS23]). *Let k, ℓ, Q be positive integers, χ_1 and χ_2 be probability distributions over R_q, and $\mathcal{C}$ be a subset of R_q. The Hint-MLWE problem* $\mathsf{H\text{-}MLWE}_{k,\ell,\chi_1,\chi_2,Q}$ *then asks an adversary $\mathcal{A}$ to distinguish between the following two cases:*

1. $(\mathbf{A}, \mathbf{As}, (c_i, \mathbf{z}_i)_{i\in[Q]})$ *for* $\mathbf{A} \leftarrow R_q^{k\times(\ell+k)}$,
2. $(\mathbf{A}, \mathbf{b}, (c_i, \mathbf{z}_i)_{i\in[Q]})$ *for* $\mathbf{A} \leftarrow R_q^{k\times(\ell+k)}$, $\mathbf{b} \leftarrow R_q^k$,

where $\mathbf{s} \leftarrow \chi_1^{\ell+k}$, $c_i \leftarrow \mathcal{C}$ *for* $i \in [Q]$, *and* $\mathbf{z}_i := c_i \cdot \mathbf{s} + \mathbf{y}_i$ *where* $\mathbf{y}_i \leftarrow \chi_2^{\ell+k}$ *for* $i \in [Q]$. *We denote by* $\epsilon_{\mathsf{H\text{-}MLWE}}$ *the advantage of $\mathcal{A}$ in solving* $\mathsf{H\text{-}MLWE}_{k,\ell,\chi_1,\chi_2,Q}$. *$\mathcal{A}$ has advantage* $\epsilon_{\mathsf{H\text{-}MLWE}}$ *in solving* $\mathsf{H\text{-}MLWE}_{k,\ell,\chi_1,\chi_2,Q}$ *if*

$$\left| \Pr\left[b = 1 \mid \begin{array}{c} \mathbf{A} \leftarrow R_q^{k\times(\ell+k)}; \mathbf{s} \leftarrow \chi_1^{\ell+k}; c_i \leftarrow \mathcal{C}; \\ \mathbf{y}_i \leftarrow \chi_2^{\ell+k}; z_i := c_i\mathbf{s} + \mathbf{y}_i \text{ for } i \in [Q]; \\ b \leftarrow \mathcal{A}(\mathbf{A}, \mathbf{As}, (c_i, \mathbf{z}_i)_{i\in[Q]}) \end{array} \right] \right.$$
$$\left. - \Pr\left[b = 1 \mid \begin{array}{c} \mathbf{A} \leftarrow R_q^{k\times(\ell+k)}; \mathbf{b} \leftarrow R_q^k; \\ \mathbf{s} \leftarrow \chi_1^{\ell+k}; c_i \leftarrow \mathcal{C}; \mathbf{y}_i \leftarrow \chi_2^{\ell+k} \\ z_i := c_i\mathbf{s} + \mathbf{y}_i \text{ for } i \in [Q]; \\ b \leftarrow \mathcal{A}(\mathbf{A}, \mathbf{b}) \end{array} \right] \right| \geq \epsilon_{\mathsf{MLWE}}.$$

Contents

Basic Signature Scheme

The private key is a short $\mathbf{s} \in R_q^{\ell+k}$, and the verification key consists of a matrix $\bar{\mathbf{A}} := [\mathbf{A} \,|\, \mathbf{I}] \in R_q^{k \times (\ell+k)}$ and vector $\boldsymbol{y} := \bar{\mathbf{A}}\mathbf{s}$. The protocol proceeds as follows:

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3. The prover responds with a short vector $\boldsymbol{z} := c \cdot \mathbf{s} + \boldsymbol{r}$.
4. $\rightarrow$The prover might abort because of rejection sampling.
5. The verifier accepts iff $\boldsymbol{z}$ is short and $\bar{\mathbf{A}}\boldsymbol{z} = c \cdot \boldsymbol{y} + \boldsymbol{w}$.
6. $\rightarrow$ Non-interactive signature scheme if $c = H(\mathsf{pk}, \boldsymbol{w}, m)$.

Basic n-out-of-n Threshold Scheme

The ith signer holds short vector $\mathbf{s}_i$ where $\mathbf{s} = \sum_{i \in [n]} \mathbf{s}_i$ is the private key. Then, the n signers can run a distributed, two-round signing protocol as follows:

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- Shamir secret shared elements are uniformly random

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- There are schemes computing threshold signatures using generic FHE (heavy computation and evaluates the RO circuit) or MPC (many rounds of commutation and distributed rejection sampling), but these are not ideal.
- We need a homomorphism to share and combine secrets, but we want to evaluate the random oracle on public input (communicated messages).

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BGV Encryption

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- $\mathsf{Dec}_{\mathsf{BGV}}$: On input a secret key $\mathsf{sk} = s$ and a ciphertext (u, v), output the message $m := (v - su \bmod q) \bmod p$.

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4. $\rightarrow$ The protocol can be made actively secure with commitments and ZKPs.

BGV DistKeyGen

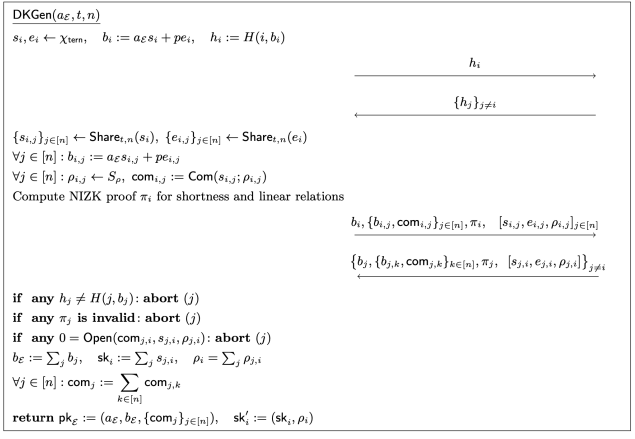

Fig. 2. Actively secure key-generation protocol, from the point of view of $\mathcal{P}_i$. The elements in square brackets with subscript j are sent to $\mathcal{P}_j$ over a private channel.

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- $\rightarrow$ TDec can be made actively secure using commitments and ZKPs.

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Ideas

- Use noise drowning techniques to avoid rejection sampling

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- Use linearly homomorphic encryption to combine shares

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- Use noise drowning techniques to avoid rejection sampling
- Use linearly homomorphic encryption to combine shares
- Use t-out-of-n threshold decryption to reconstruct signatures

Passive Protocol

Keys $\mathbf{s}$ and $(\bar{\mathbf{A}}, \boldsymbol{y} := \bar{\mathbf{A}}\mathbf{s})$ are as before. Instead of sharing $\mathbf{s}$, signers will hold an encryption $\mathsf{ctx}_{\mathbf{s}} = \mathsf{Enc}(\mathbf{s})$ and share the decryption key $\boldsymbol{k}$ in a t-out-of-n fashion:

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2. Each signer computes $\boldsymbol{w} := \sum_{i \in \mathcal{U}} \boldsymbol{w}_i$, $c = H(\boldsymbol{w})$, and "encrypted signature" $\mathsf{ctx}_{\boldsymbol{z}} := c \cdot \mathsf{ctx}_{\mathbf{s}} + \sum_{i \in \mathcal{U}} \mathsf{ctx}_{\mathbf{r}_i}$. It sends its threshold decryption share of $\mathsf{ctx}_{\boldsymbol{z}}$.

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3. Given decryption shares from all parties, each signer can decrypt $\mathsf{ctx}_{\boldsymbol{z}}$ to obtain $\boldsymbol{z}$, and output the signature $(c, \boldsymbol{z})$.

Passive Protocol

$\underline{\mathsf{Sign}_{\mathcal{TS}}(\mathsf{sk}_i, \mathcal{U}, \mu)}$

sample bounded $\boldsymbol{r}_i \leftarrow D_r$

$\boldsymbol{w}_i := \bar{\mathbf{A}}\boldsymbol{r}_i, \quad \mathsf{ctx}_{\boldsymbol{r}_i} := \mathsf{Enc}(\mathsf{pk}_{\mathcal{E}}, \boldsymbol{r}_i)$ $\xrightarrow{\boldsymbol{w}_i, \mathsf{ctx}_{\boldsymbol{r}_i}}$

$\boldsymbol{w} := \sum_{j \in \mathcal{U}} \boldsymbol{w}_j, \quad c := H(\boldsymbol{w}, \mathsf{pk}, \mu)$ $\xleftarrow{\{(\boldsymbol{w}_j, \mathsf{ctx}_{\boldsymbol{r}_j})\}_{j \in \mathcal{U} \setminus \{i\}}}$

$\mathsf{ctx}_{\boldsymbol{z}} := c \cdot \mathsf{ctx}_{\mathbf{s}} + \sum_{j \in \mathcal{U}} \mathsf{ctx}_{\boldsymbol{r}_j}$

$\mathsf{ds}_i := \mathsf{TDec}(\mathsf{ctx}_{\boldsymbol{z}}, \mathsf{sk}_i, \mathcal{U})$ $\xrightarrow{\mathsf{ds}_i}$

$\boldsymbol{z} := \mathsf{Comb}(\mathsf{ctx}_{\boldsymbol{z}}, \{\mathsf{ds}_j\}_{j \in \mathcal{U}})$ $\xleftarrow{\{\mathsf{ds}_j\}_{j \in \mathcal{U} \setminus \{i\}}}$

return $\sigma := (c, \boldsymbol{z})$

Contents

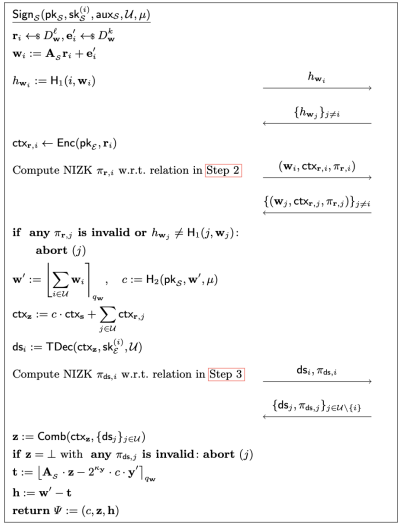
Optimizations

- Renyi Divergence instead of noise drowning for decryption
- MLWE with Hints instead of noise drowning for signatures
- Three round scheme $\rightarrow$ avoid trapdoor commitments
- Pre-processing first two rounds $\rightarrow$ non-interactive signing
- Using improved zero-knowledge proofs from the literature
- Improved distributed key generation for many parties
- Formalizing and proving identifiable aborts for our signatures
- Implementation based on the Raccoon signature scheme

Distributed Key Generation

$$\begin{bmatrix} \lambda_1^{(1)} & \cdots & \lambda_{t-1}^{(1)} & \lambda_t^{(1)} & 0 & \cdots & 0 & -1 \\ \lambda_1^{(2)} & \cdots & \lambda_{t-1}^{(2)} & 0 & \lambda_{t+1}^{(2)} & 0 & \cdots & -1 \\ \vdots & & \vdots & & & \ddots & & \vdots \\ \lambda_1^{(n-t+1)} & \cdots & \lambda_{t-1}^{(n-t+1)} & 0 & \cdots & 0 & \lambda_n^{(n-t+1)} & -1 \end{bmatrix} \begin{bmatrix} [\![\mathbf{S}]\!]_1 \\ [\![\mathbf{S}]\!]_2 \\ \vdots \\ [\![\mathbf{S}]\!]_n \\ \mathbf{S} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix}$$

Actively Secure Scheme



Contents

Comparison

Scheme	PK	SIG	COM	Rounds	Users	DKG	ID-A
[GKS24]	13.6	46.6	3000	$0+2$	5	✓	(✓) [9]
[DKM+24]	3.9	12.7	41	$0+3$	1024	✗	✗
[EKT24] [10]	5.5	10.8	538	$1+1$	1024	✗	✗
[EKT24, ZT25]	8.7	30.9	767	$1+1$	1024	✗	✗
[KRT24]	3.9	12.7	—	$2+3$	1024	✗	✗
[CATZ24, ZT25]	42.1	144.5	1240	$1+1$	32	✗	(✗) [11]
[BKL+25]	4.5	13.4	629	$1+1$	1024	✗	✗
This work	2.6	9.7	570	$2+1$	1024	✓	✓

Thank you! Questions?