Zero Knowledge Proofs:
Challenges, Applications, and Real-world Deployment

NIST Workshop on Privacy Enhancing Cryptography

September 26th, 2024

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AI Research

This talk

1) Introduction to Zero Knowledge Proof (Akira)

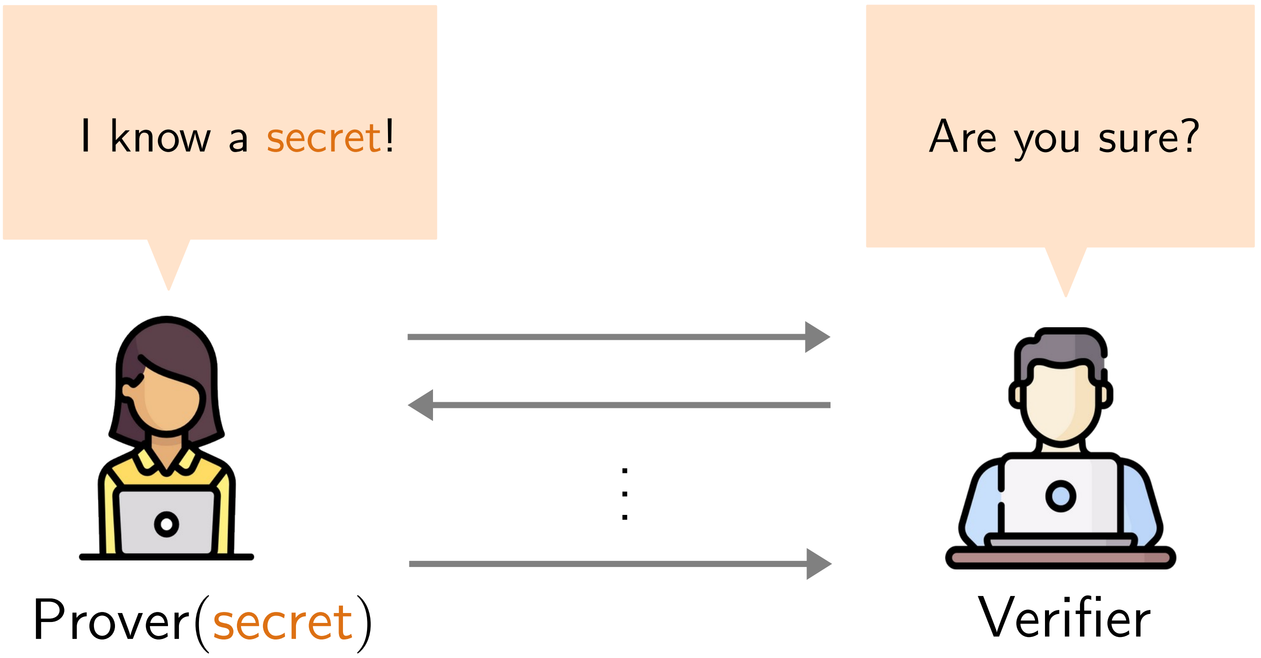
2) Technical Challenges (Akira)

3) Real-World Applications (Tjerand)

4) Insights from ZKP Workshop (Tjerand)

5) Resources and Standards (Tjerand)

What is Zero Knowledge Proof?



Basics

- ZKP is a two-party protocol, consisting of **Prover** and **Verifier**
- With ZKP, Prover can convince Verifier that she has some secret information without disclosing the secret
- Long history of research starting from the ‘80s [GMR85]. Lots of efficiency improvements during the last decade
  - cf. **ZK-SNARK** (Succinct Non-interactive Argument of Knowledge)

Syntax of ZKP

- x: statement (i.e. public input)
- w: witness (i.e. secret input)
- R: relation function, outputting 1 or 0
- "I know w s.t. $R(x, w) = 1$"

b: decision bit

Completeness

- If Prover and Verifier honestly follow the protocol, then Verifier halts by outputting $b = 1$

Prover(x, w)

Verifier(x)

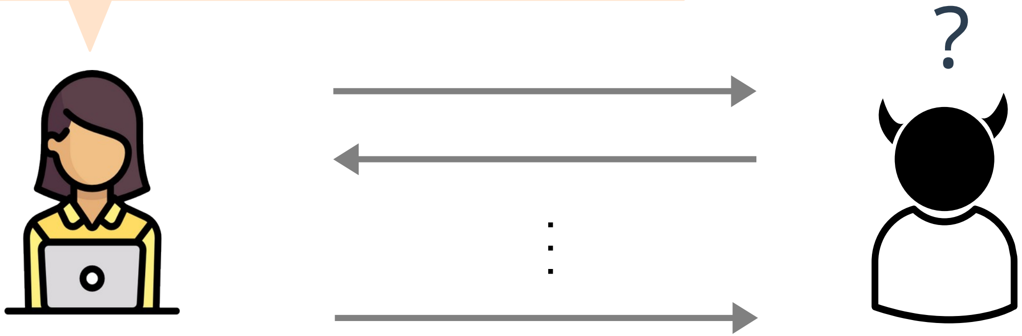
$b = 1$: "accept"

$b = 0$: "reject"

Security Goals of Zero Knowledge Proof

- x: statement (i.e. public input)
- w: witness (i.e. secret input)
- R: relation function, outputting 1 or 0
- "I know w s.t. $R(x, w) = 1$"

- Tries to steal w



Prover(x, w) Verifier(x)

Zero Knowledge (ZK)

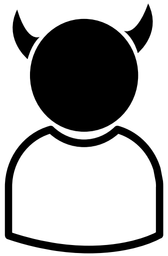
- Protecting against malicious verifier
- Verifier learns nothing about Prover's secret
- Formally, ZK is guaranteed by showing the existence of "Simulator"

Security Goals of Zero Knowledge Proof

- Maliciously interacts with Verifier
- Might use an "invalid" input

$$R(x^*, w^*) = 0$$

You lied!



Prover(x^*, w^*)

Verifier(x^*)

$b = 0$: "reject"

Zero Knowledge (ZK)

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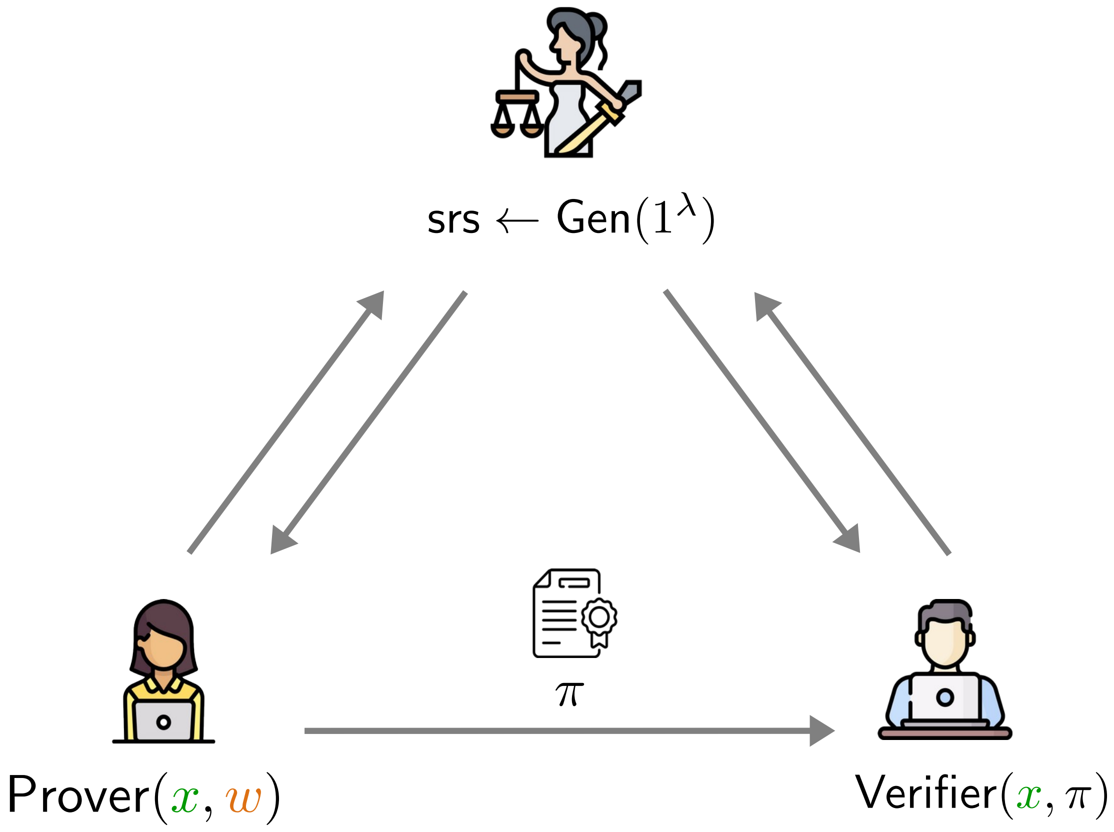
Knowledge Soundness (KSND)

- Protecting against malicious prover
- If Prover uses an invalid secret, then Verifier catches it with high probability
- Formally, knowledge soundness is guaranteed by showing the existence of "Knowledge Extractor"

Non-interactive Zero Knowledge Proof (NIZK)



Removing Interactions

- Ideally, Prover should create a one-shot proof string π
- Verifier checks π asynchronously
- Such π is reusable and can be checked by potentially many verifiers

Types of Trusted Setup

- Structured Reference String (SRS)
- Hash function modeled as Random Oracle
- Or both!

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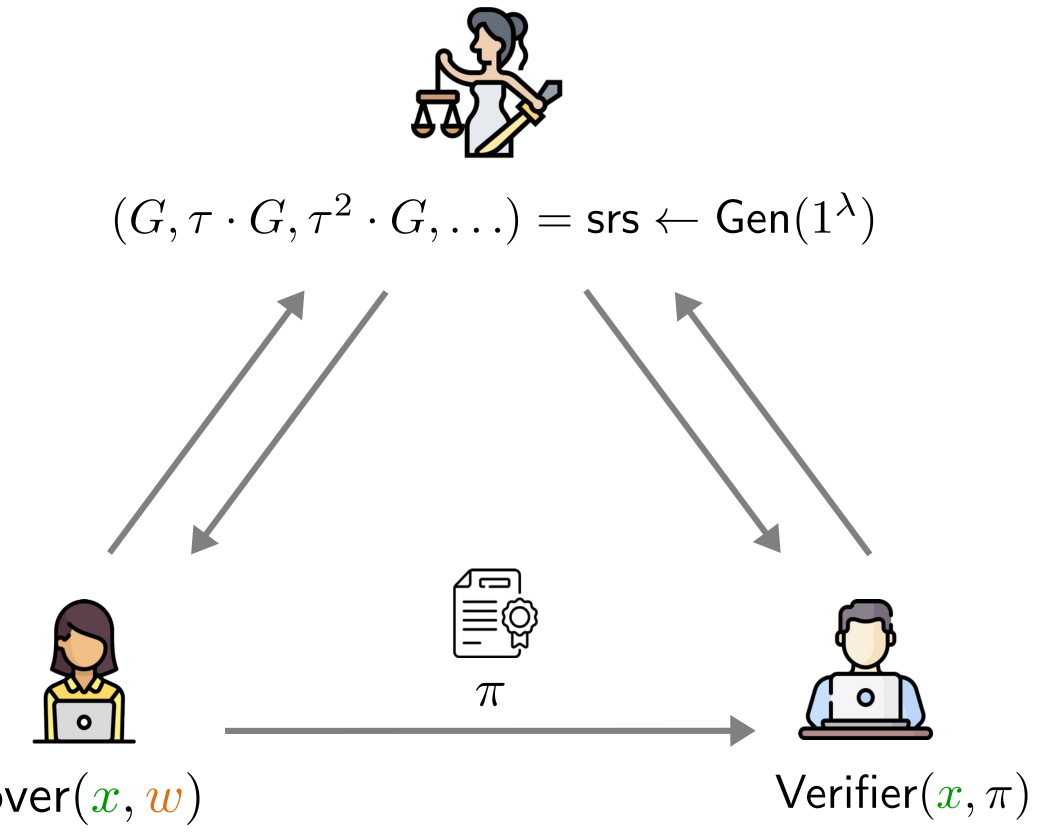

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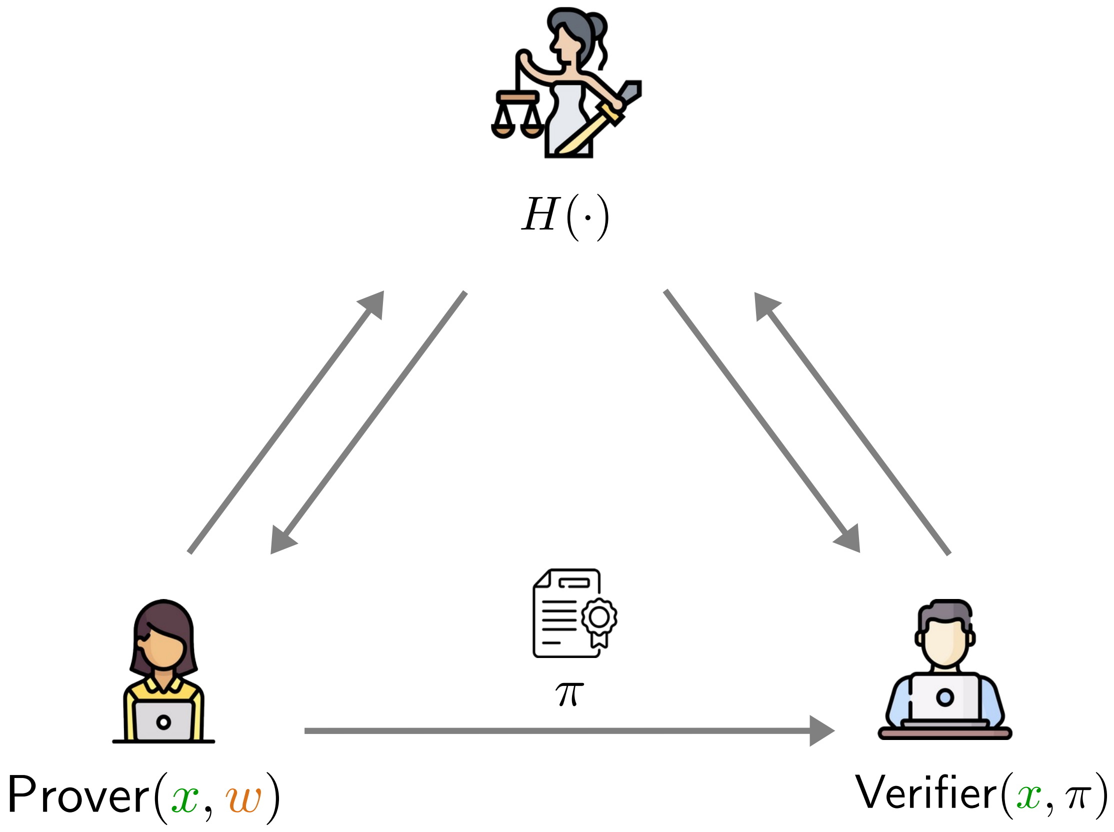

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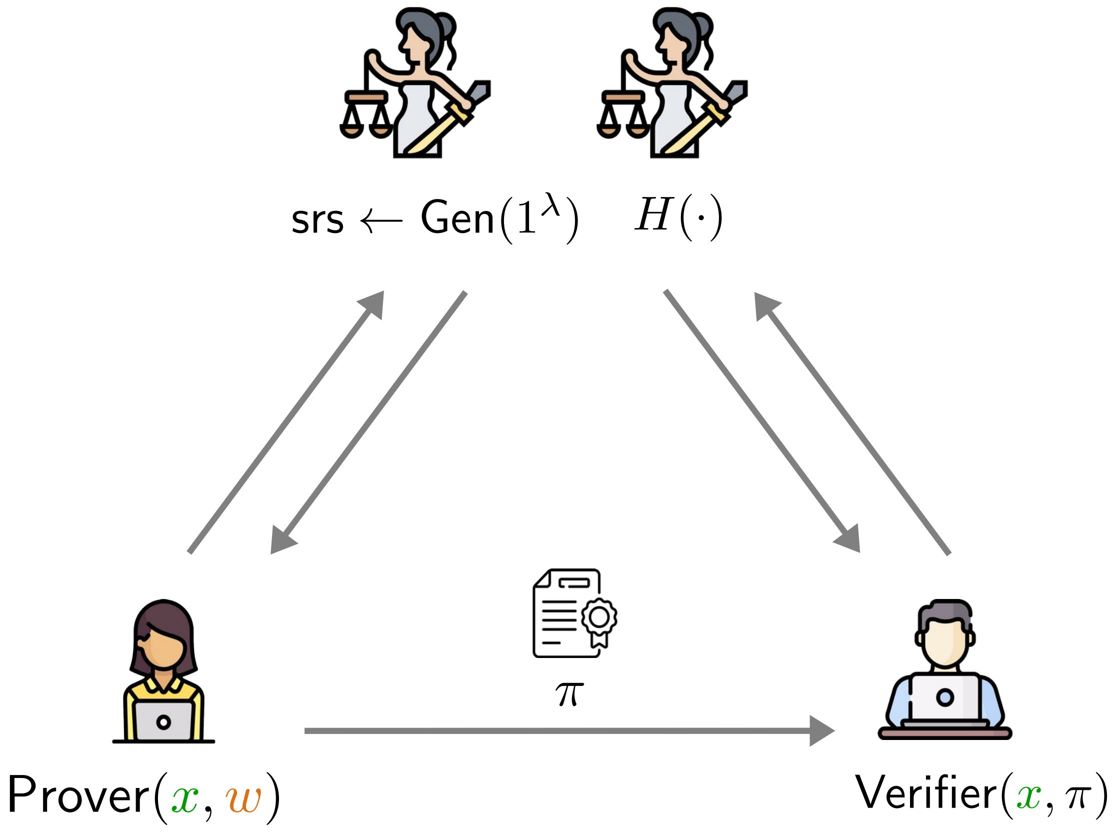

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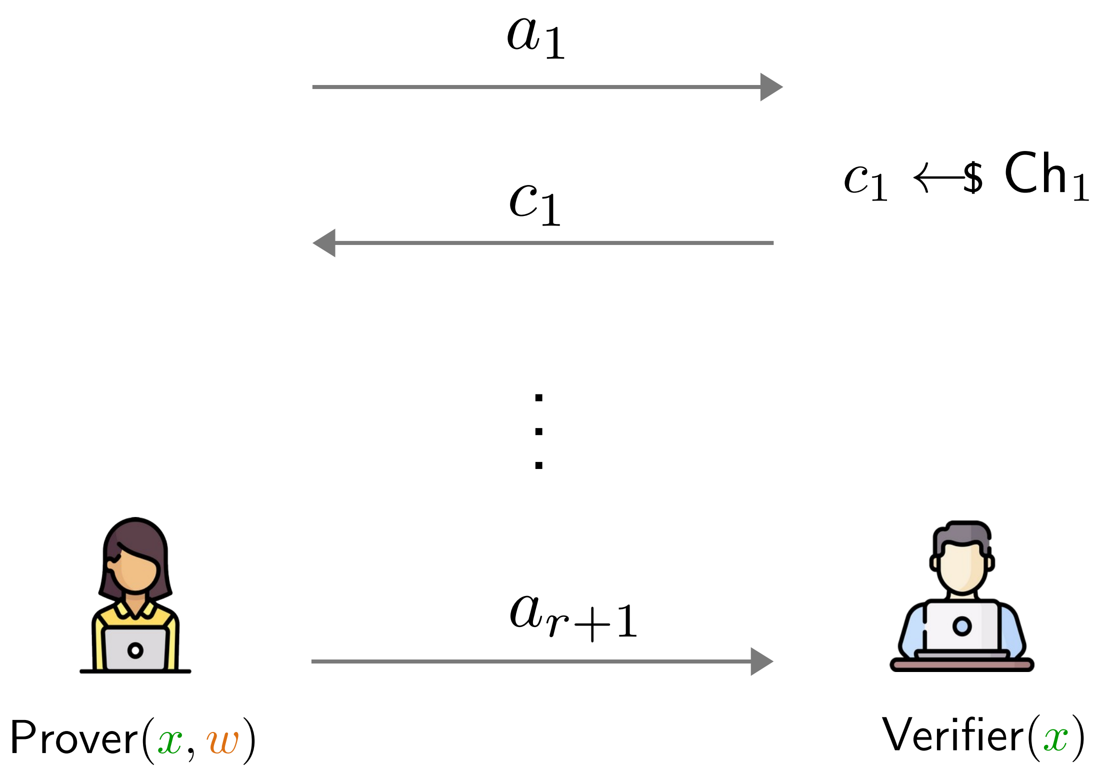
Paradigm of NIZK I: Fiat-Shamir [FS87]



Modular Design of NIZK

- **Step 1.** Construct a "public-coin" interactive protocol
  - Verifier does not require a secret state
  - ZK against semi-honest Verifier (Honest-Verifier ZK)
- **Step 2.** NI Prover and Verifier obtain challenge by locally hashing a partial transcript so far
- Bonus: By hashing the message, FS-NIZK gives rise to a digital signature
- Example: Schnorr/EdDSA, CRYSTALS-Dilithium, PLONK family, Bulletproofs, etc.
- Many modern SNARKs are constructed from (Polynomial) Interactive Oracle Proofs converted to NIZK via Fiat-Shamir [BCS16, CHMMVW19, BFS19, GWC19, CFFQR20,...]

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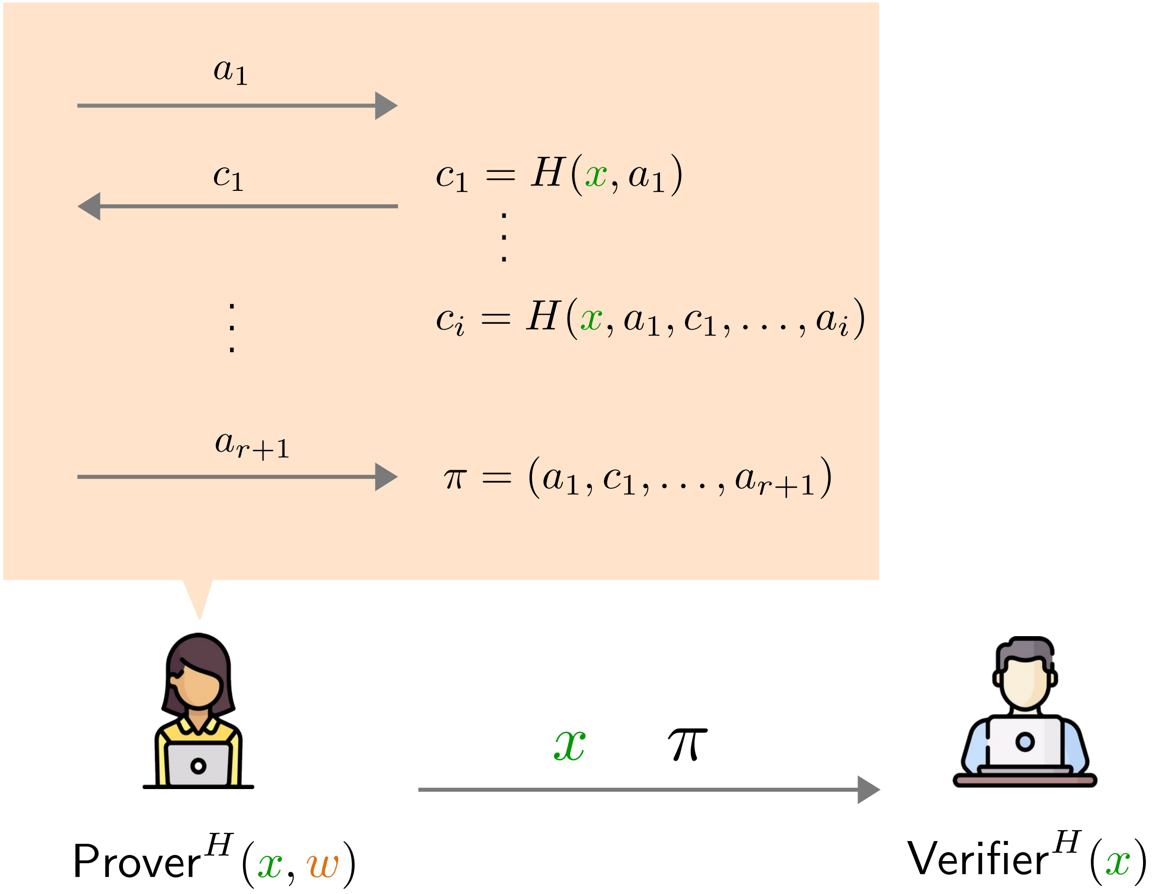

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Paradigm of NIZK I: Fiat-Shamir [FS87]

a_1 →

c_1 ← $\quad c_1 = H(vk, a_1, \text{msg})$

$\vdots$

$c_i = H(vk, a_1, c_1, \ldots, a_i, \text{msg})$

a_{r+1} → $\quad \sigma = (a_1, c_1, \ldots, a_{r+1})$

vk, msg, σ →

$\text{Signer}^H(vk, sk, \text{msg})$ $\qquad$ $\text{Verifier}^H(vk, \sigma, \text{msg})$

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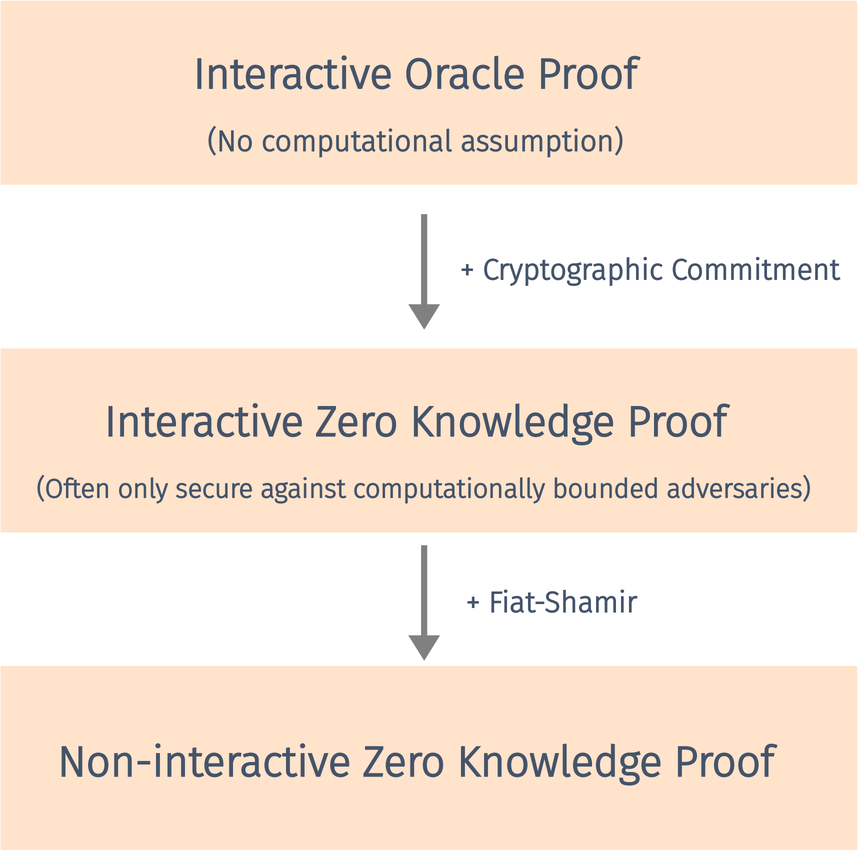

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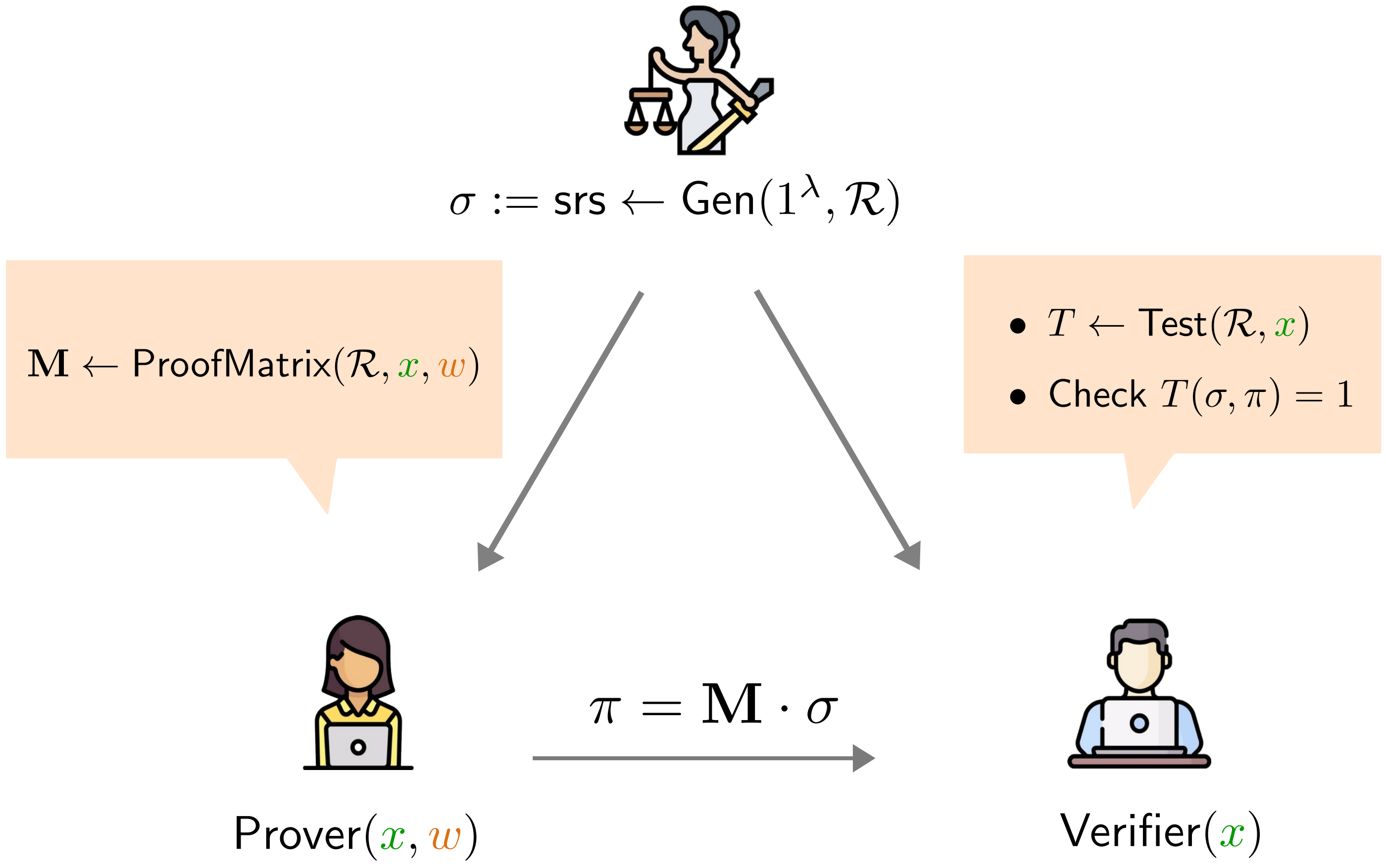
Paradigm of NIZK II: Linear Interactive Proofs [GGPR13,BCI+13]



NIZK without Fiat-Shamir

- **Step 1.** srs generator outputs a relation-dependent vector
- **Step 2.** NI Prover applies linear transformation to srs
- **Step 3.** NI Verifier derives a testing function, allowing to check whether correct linear transformation has been applied
- Example: Groth16
- Important: Prover and Verifier should never learn internal randomness of Gen; otherwise, malicious prover can easily prove a false statement

Technical Challenges

1) Balancing Generality, Efficiency and Assumptions

2) Advanced Security

3) Interoperability

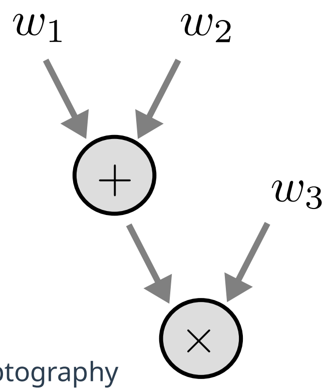
Types of ZKP

General-Purpose ZKP

- Supports arbitrary NP relation R
- Relation is often described using an arithmetic circuit

$$\mathcal{R}_C = \{(x, w) : C(x, w) = 1\}$$

- Pros:
  - Can prove correct execution of *any* program
  - Suitable for verifiable and outsourced computation
- Cons:
  - circuit gets complex for certain non-linear computations
  - E.g., elliptic curve arithmetic, comparison, table lookup, etc.



Specialized ZKP

- Designed for particular type of NP relation R

$$\mathcal{R}_{\mathrm{DL}} = \{(X, w) : X = w \cdot G\}$$

$$\mathcal{R}_{\mathrm{SIS}} = \{(\mathbf{x}, \mathbf{w}) : \mathbf{x} = \mathbf{A}\mathbf{w} \bmod q, \|\mathbf{w}\| \leq \beta\}$$

$$\mathcal{R}_{\mathrm{Lookup}} = \{(\mathbf{x}, \mathbf{w}) : \mathbf{w} \text{ is a subvector of } \mathbf{x}\}$$

- Pros:
  - Can prove and verify designated relations efficiently
  - Sufficient for some useful applications, e.g., proof of correct encryption, distributed key generation, signatures, etc.
- Cons:
  - Requires careful integration with general-purpose ZKP to support more complex statements

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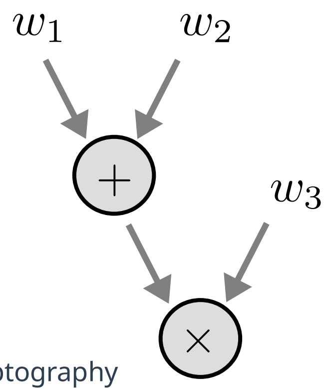

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Desiderata

Proof Size

- Smaller proof saves storage and communication bandwidth
- Groth16 requires *only 3 group elements* from pairing-friendly curves
- State-of-the-art Polymath [Lip24] and PARI [DMS24] achieve even smaller proof sizes!

Assumptions

- To minimize a trust assumption, SRS should be avoided
- Better alternative: only trust the security of hash function modeled as RO (aka transparent setup), e.g., Bulletproofs, Brakedown, STARK, LaBRADOR, MPC/VOLE-in-the-Head, etc.
- Middle-ground solution: allows different parties to update SRS (aka updatable SRS) [GKMMM18]

Setup, Prover and Verifier Cost

- Universal Setup: Setup outputs SRS once and for all for arbitrary circuits

$$\mathsf{srs} \leftarrow \mathsf{Setup};\ \mathsf{srs}_C \leftarrow \mathsf{Derive}(\mathsf{srs}, C)$$

- Verifier sub-linear in $|C|$
- Prover time linear in #non-linear gates

Scalability

- How can we prove a large statement efficiently?
  - **Proof Aggregation**: aggregate many, asynchronously generated proofs, e.g., SnarkPack
  - **Incrementally Verifiable Computation [Valiant08]**: succinct proof of incremental computations via recursion or folding, e.g., Halo2, Nova, etc.

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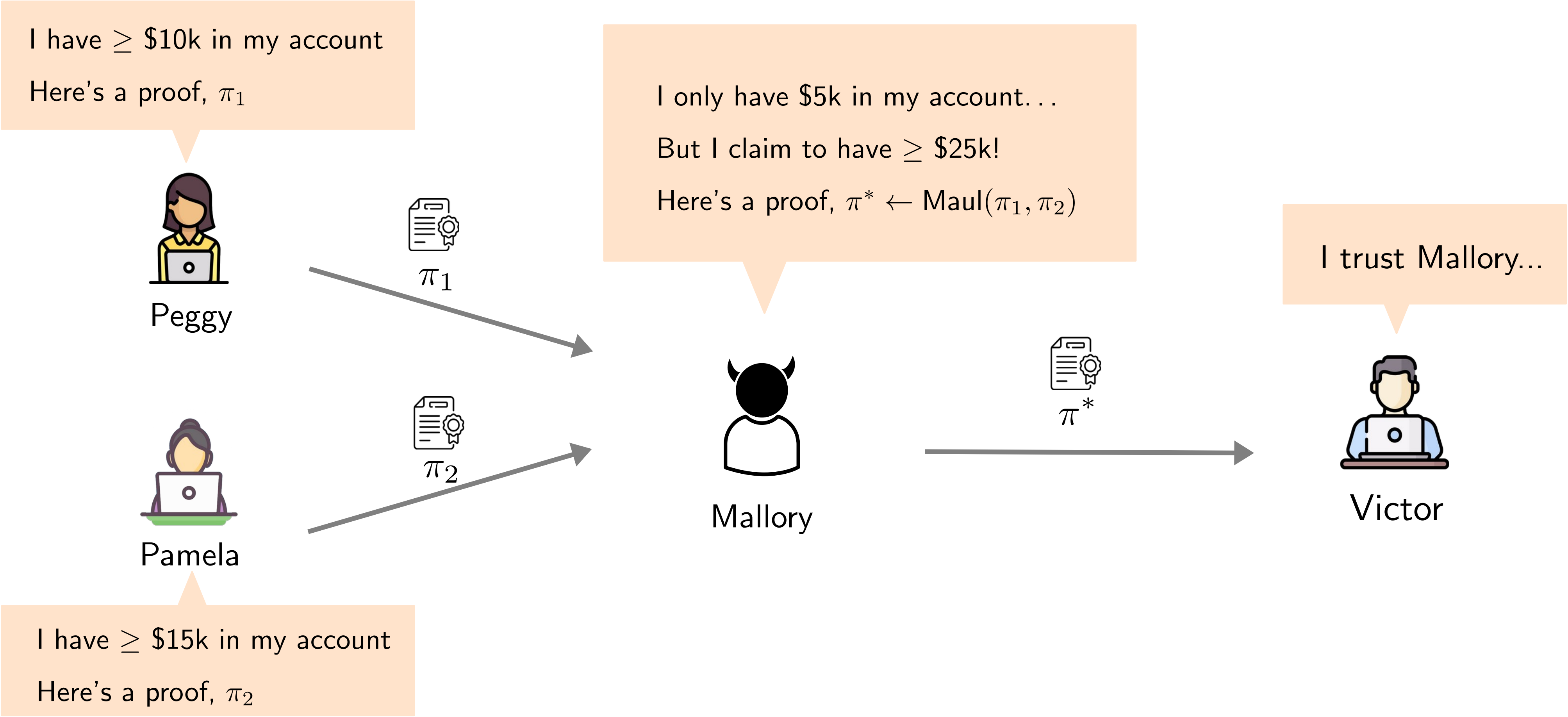
Technical Challenges

1) Balancing Generality, Efficiency and Assumptions

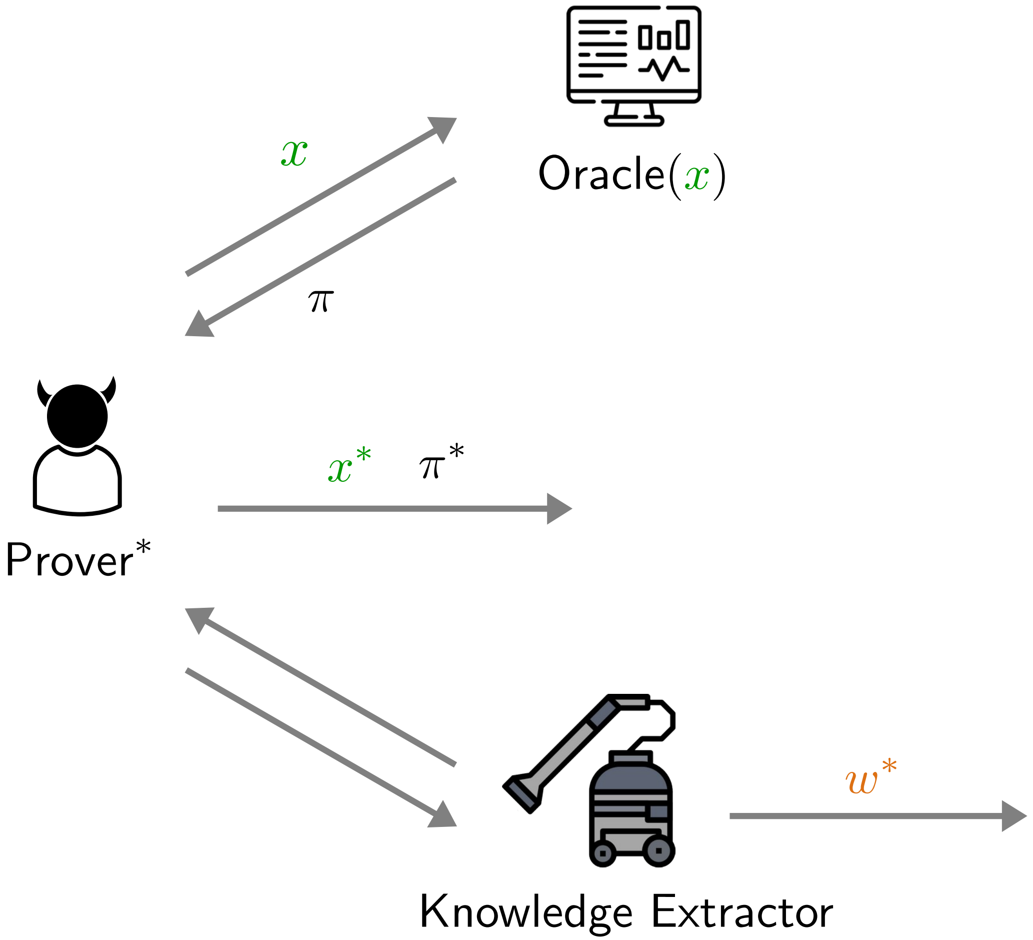
2) Advanced Security

3) Interoperability

ZK and Knowledge Soundness are not Enough: Malleability Attacks



Combined Notion: Simulation-Extractability



SIM-EXT Security

1. Prover* obtains fresh proof from Oracle
2. Prover* outputs "forgery" (x^*, π^*)
3. If (x^*, π^*) is accepting and not recorded by Oracle, then Prover* must know the corresponding witness w^*

- Intuitively, SIM-EXT guarantees **non-malleability:** a cheating prover cannot maul existing proofs to create a new one, without knowing a valid witness
- Cf. (S)EUF-CMA for signature and IND-CCA for PKE
- Crucial property NIZK should satisfy if used as a subroutine of another protocol
- Many practical NIZK schemes turn out to be SIM-EXT [GKKNZ22] [GOPTT22] [DG23] [FFKR23] [KPT23] [Lib24] [FFR24]
- Some schemes satisfy UC security [Canetti01] accepting some idealized setup [CF24] [BFKT24]

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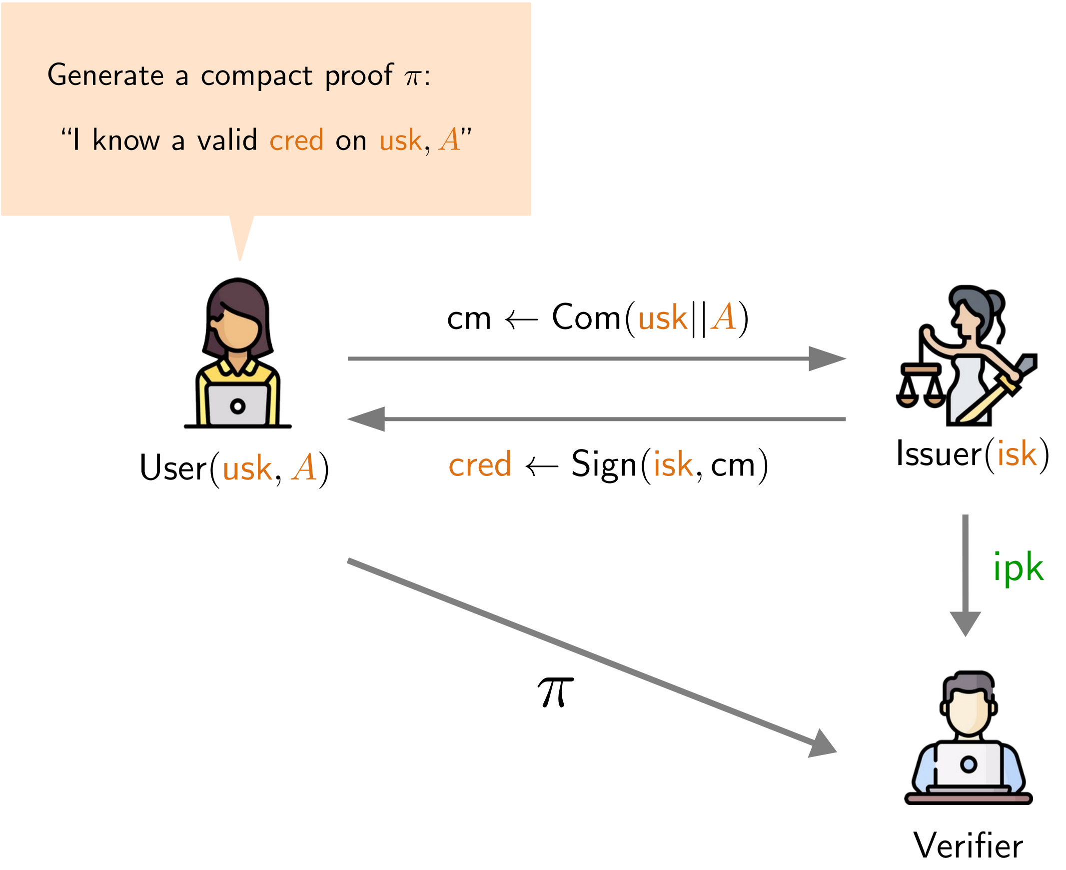
Anonymous Credentials (High Level)



Protocol

- Issuer initially binds attributes and usk to secret credentials
- The owner of attributes produces a **proof string** in the form of ZKP
- By examining the proof string, Verifier gets convinced that User has valid attributes signed by Issuer
- Thanks to ZKP, the proof string only leaks minimum info about Prover's identity
- E.g., Verifier learns "User is => 21 years old" but nothing else

- isk: issuer secret key
- ipk: issuer public key
- usk: user secret key
- A: user attributes

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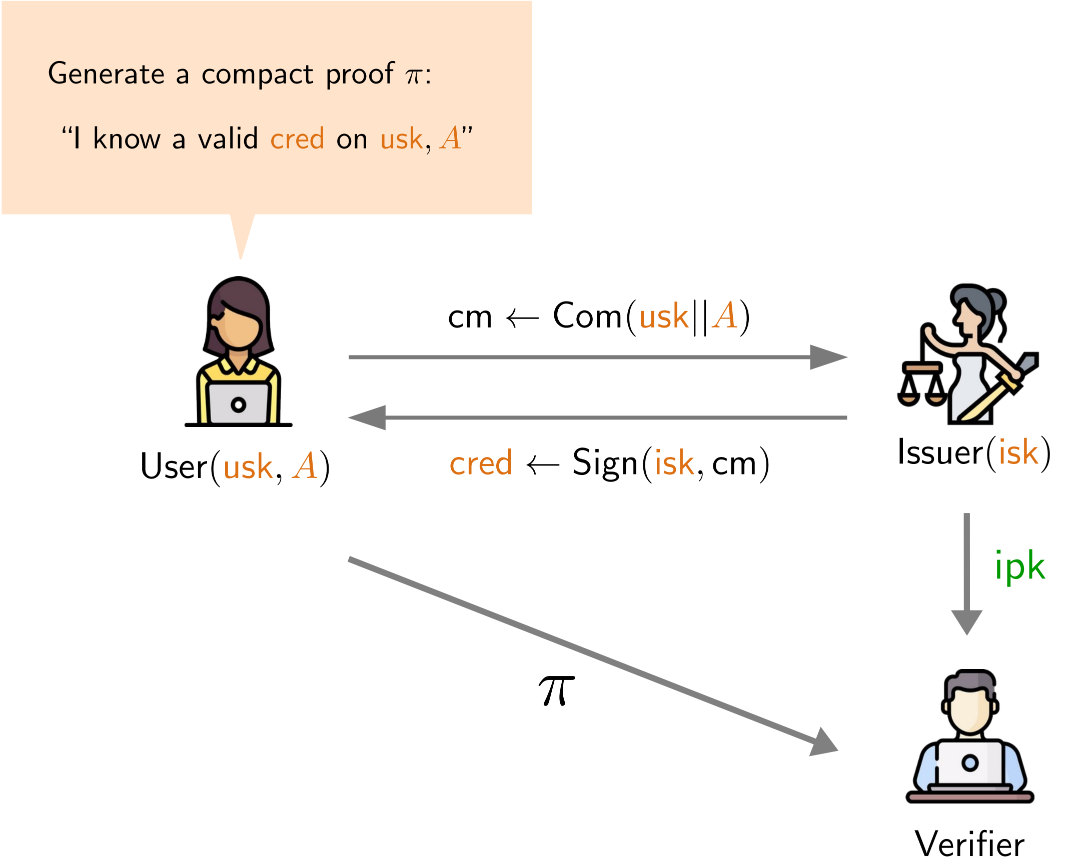

Interoperability

- Central ZKP for AC: Proof-of-Knowledge of valid signature
- If an arbitrary signature scheme is allowed, many efficient solutions exist: BBS+signature
- However, interoperability with standardized and widely deployed signature is often preferred in practice, e.g., RSA-PSS, ECDSA, EdDSA, etc.
- Verification condition of deployed schemes are not very ZK friendly. Can we make tailored ZKP more efficient?

- isk: issuer secret key
- ipk: issuer public key
- usk: user secret key
- A: user attributes

Takeaways

- ZKP allows Prover to prove the knowledge of a secret, while Verifier learns nothing about the secret
- Important Security Properties: **Knowledge Soundness** and **Zero Knowledge**
- Choose between general-purpose ZKP and specialized ZKP, or compose them carefully
- Which setup assumption is suitable for deployment?
  - Trusted, Transparent, Updatable, …
- What should you optimize?
  - Proof Size, Setup / Prover / Verifier Costs, Scalability, Assumptions, …
- Check whether ZKP satisfies advanced security such as SIM-EXT or UC if ZKP is used a building block of another protocol
- More research needed to optimize ZKP while retaining interoperability with standardized signatures or encryption schemes

References

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