

INTRODUCTION TO THE NEW POST-QUANTUM STANDARDS

Tjerand Silde @ NDC Security 2026

Introduction

- ▶ Associate Professor in Cryptology
- ▶ Department of Information Security and Communication Technology at NTNU
- ▶ Leading NTNU Applied Cryptology Lab
- ▶ Quantum-safe cryptography, privacy applications & secure implementation
- ▶ Cryptography Expert @ Pone Biometrics



Motivation

- ▶ The public key cryptographic primitives we use today are based on the hardness of factoring and discrete logarithm assumptions
- ▶ These assumptions can (in theory) be broken by Shor's algorithm on quantum computers, and quantum computing has seen recent advances
- ▶ Symmetric key encryption and hash functions seem to be fine; in the worst case, we need to double keys and outputs due to Grover's algorithm
- ▶ We need to standardize new Key Encapsulation Mechanisms (KEM) and Digital Signatures (DS) from quantum-safe assumptions, e.g., lattices
- ▶ There are also other KEM and DS standards that are not covered in this lecture, and there is an ongoing additional DS competition as we speak

Contents

Quantum-Safe Cryptography

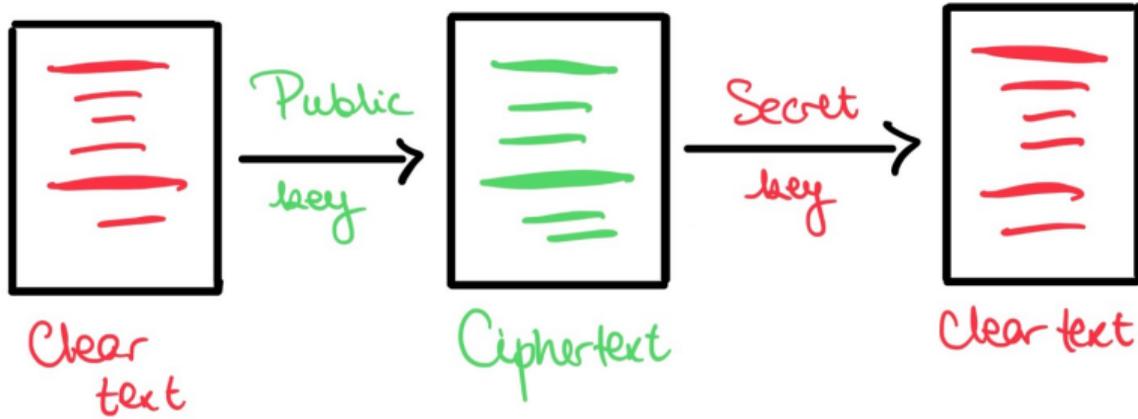
New Hardness Assumption

ML-KEM (CRYSTALS-Kyber)

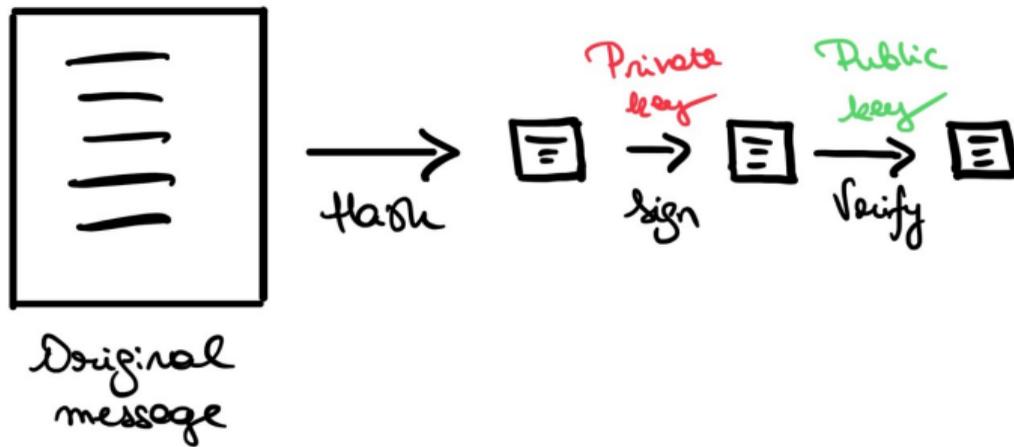
ML-DSA (CRYSTALS-Dilithium)

Standards and Implementation

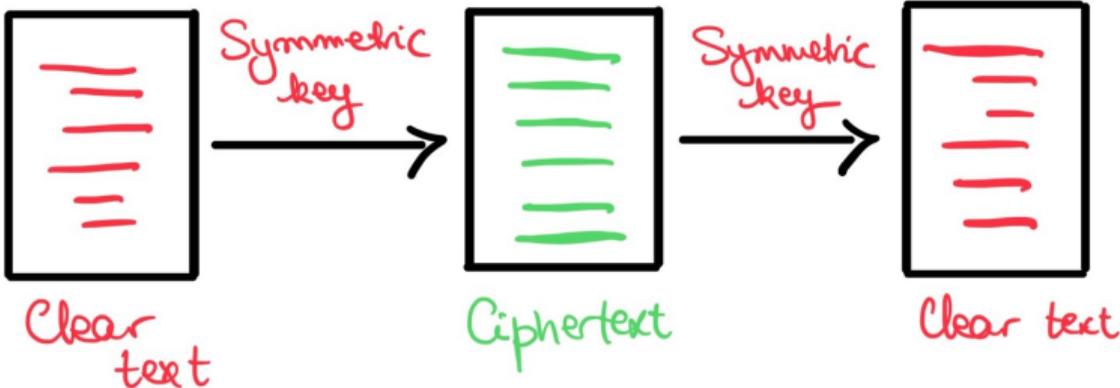
Public-key Encryption



Digital Signatures

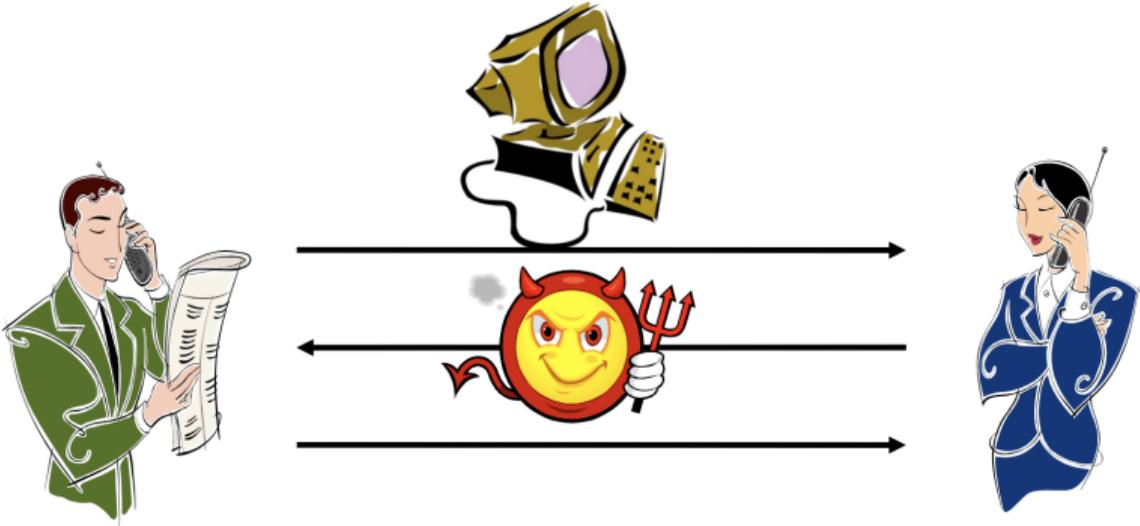


Symmetric-key Cryptography



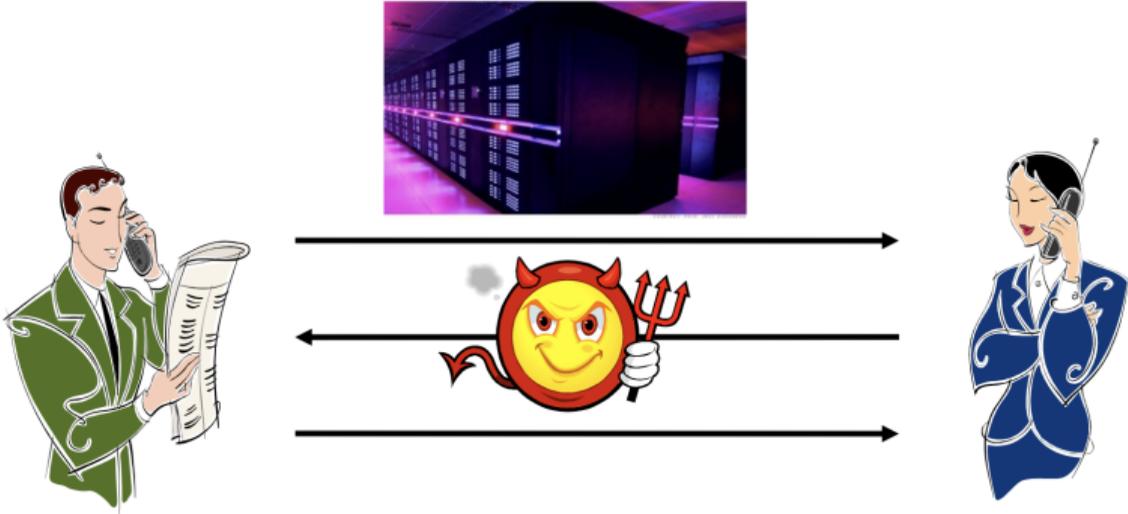
Cryptography Today

Allows for secure communication in the presence of malicious parties



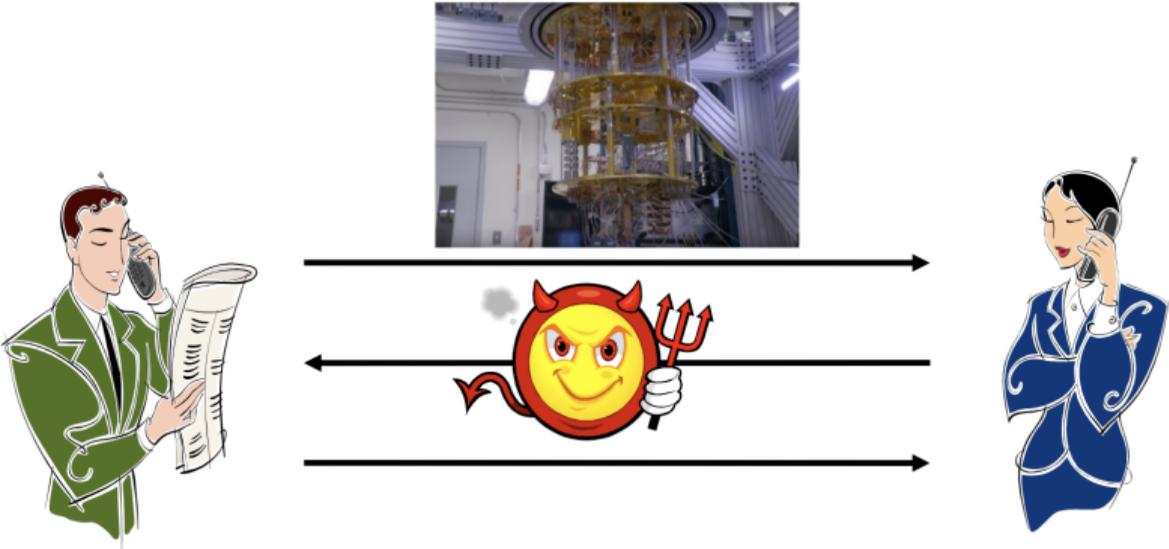
Cryptography Today

Large increase in the adversary's computing power
requires only a small increase in the key size



Cryptography Tomorrow

A quantum computer is outside the classical model of computation for efficiency purposes

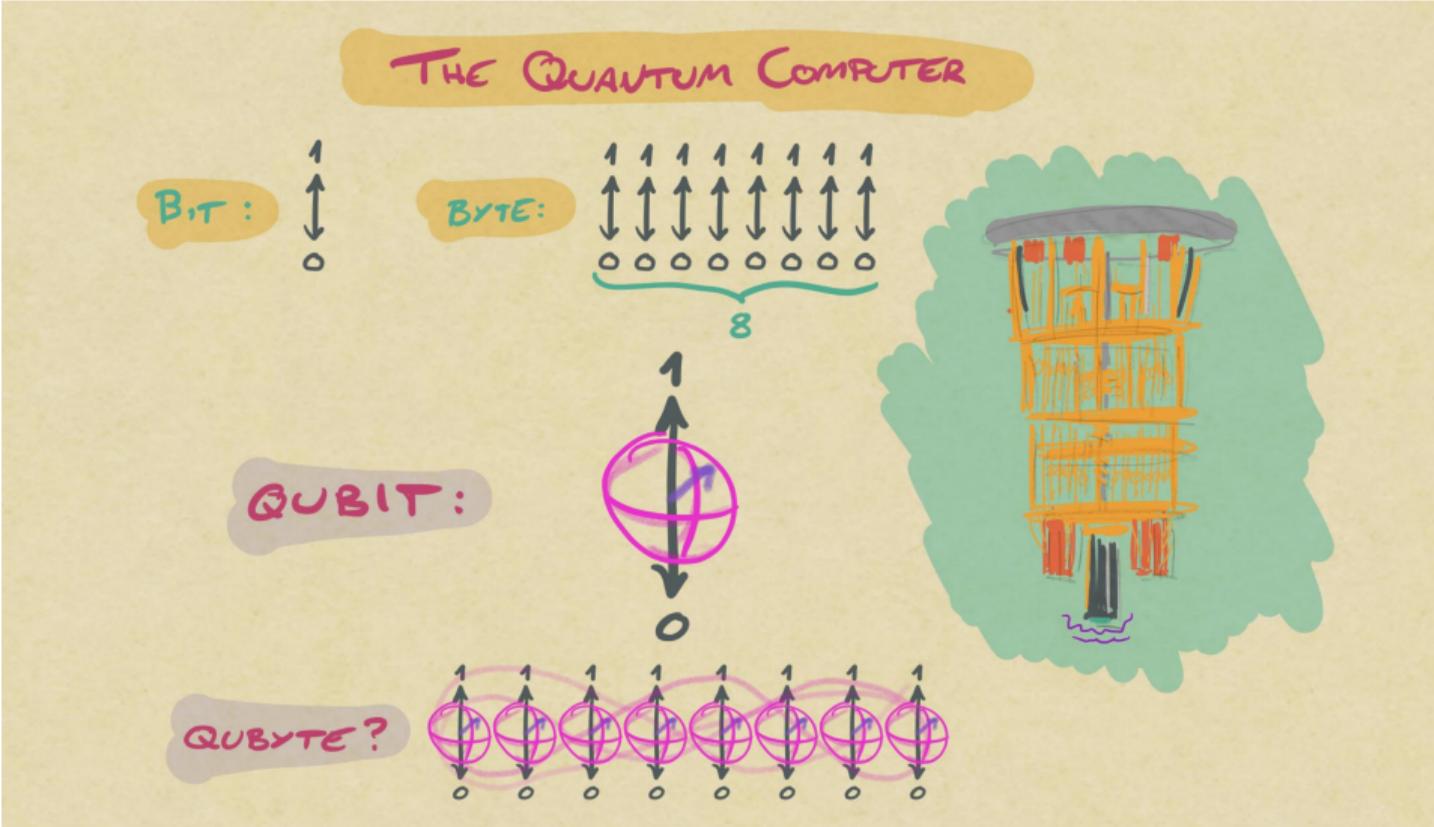


Cryptography Tomorrow

Shor's quantum algorithm can factorize integers and compute discrete logs essentially as fast as using them, given a large quantum computer. This will break RSA, (EC)DH, (EC)DSA schemes and others relying these assumptions. To achieve future secrecy, there is an urgent need to replace those algorithms.

Grover's quantum algorithm can improve exhaustive search by a square root factor, potentially speeding up key recovery and finding hash collisions.

Quantum Computers



Quantum Computers

- ▶ Quantum computers are not better; they are different
- ▶ They will generally be worse, but do specific things better
- ▶ In theory, they can break public key encryption and digital signatures based on factoring and discrete log assumptions
- ▶ There are many recent developments in quantum computing

Factoring RSA

How to factor 2048 bit RSA integers with less than a million noisy qubits

Craig Gidney

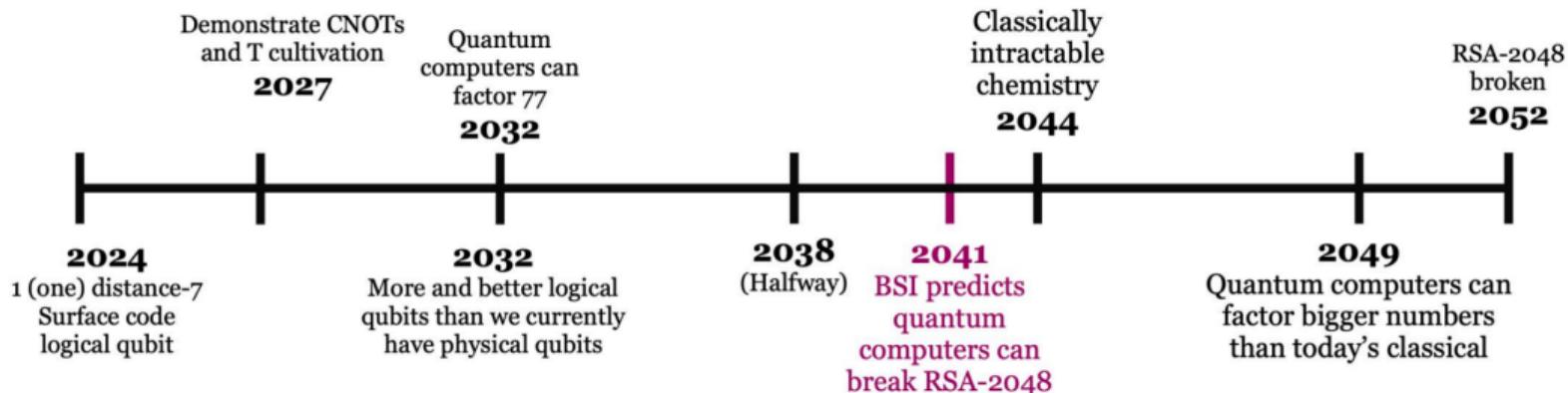
Google Quantum AI, Santa Barbara, California 93117, USA

June 9, 2025

Planning the transition to quantum-safe cryptosystems requires understanding the cost of quantum attacks on vulnerable cryptosystems. In Gidney+Ekerå 2019, I co-published an estimate stating that 2048 bit RSA integers could be factored in eight hours by a quantum computer with 20 million noisy qubits. In this paper, I substantially reduce the number of qubits required. I estimate that a 2048 bit RSA integer could be factored in less than a week by a quantum computer with less than a million noisy qubits. I make the same assumptions as in 2019: a square grid of qubits with nearest neighbor connections, a uniform gate error rate of 0.1%, a surface code cycle time of 1 microsecond, and a control system reaction time of 10 microseconds.

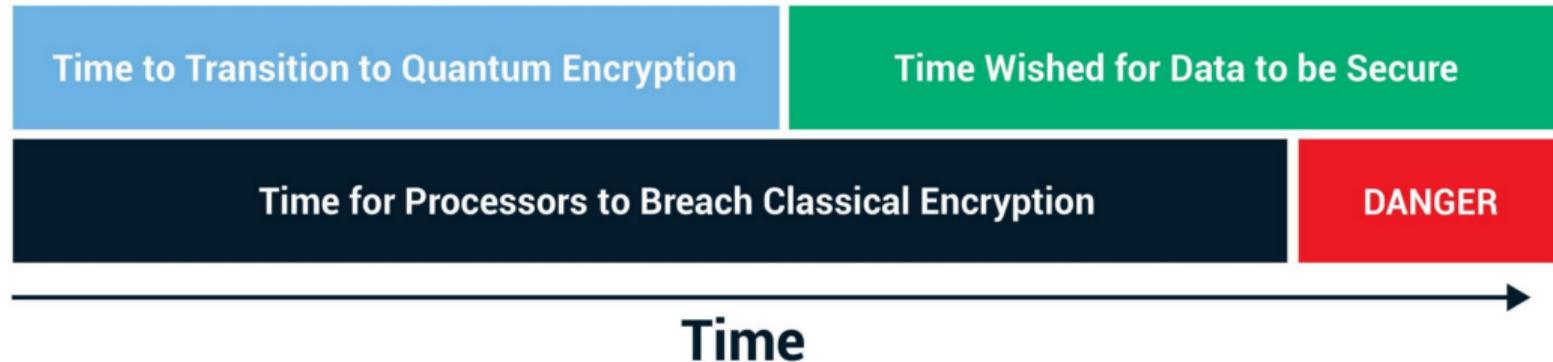
Figure: <https://arxiv.org/pdf/2505.15917>

BSI Timeline



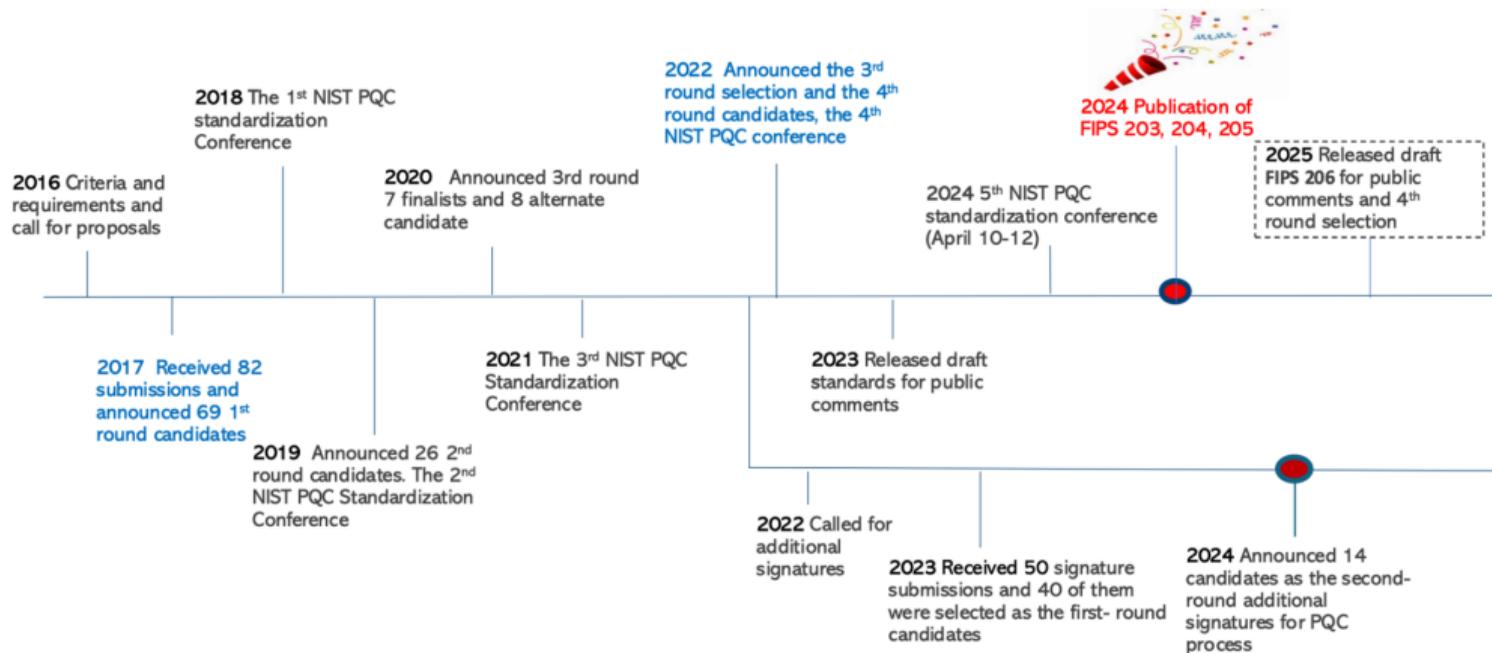
Harvest now, decrypt later

Urgency: Mosca's Inequality



Don't wait - upgrade your encryption now!

NIST Timeline





Hybrid

Quantum-resistant schemes

- Establishing a shared secret between two parties:

<i>Scheme</i>	<i>Status</i>
ML-KEM	D

ML-KEM must be used in hybrid mode with a quantum-vulnerable key establishment scheme using an appropriate KEM combiner. The recommended parameter sets are ML-KEM-768 and ML-KEM-1024.

Figure: <https://nsm.no/regelverk-og-hjelp/veiledere-og-handboker/kryptografi-ske-anbefalinger-en-veileder-fra-nsm>

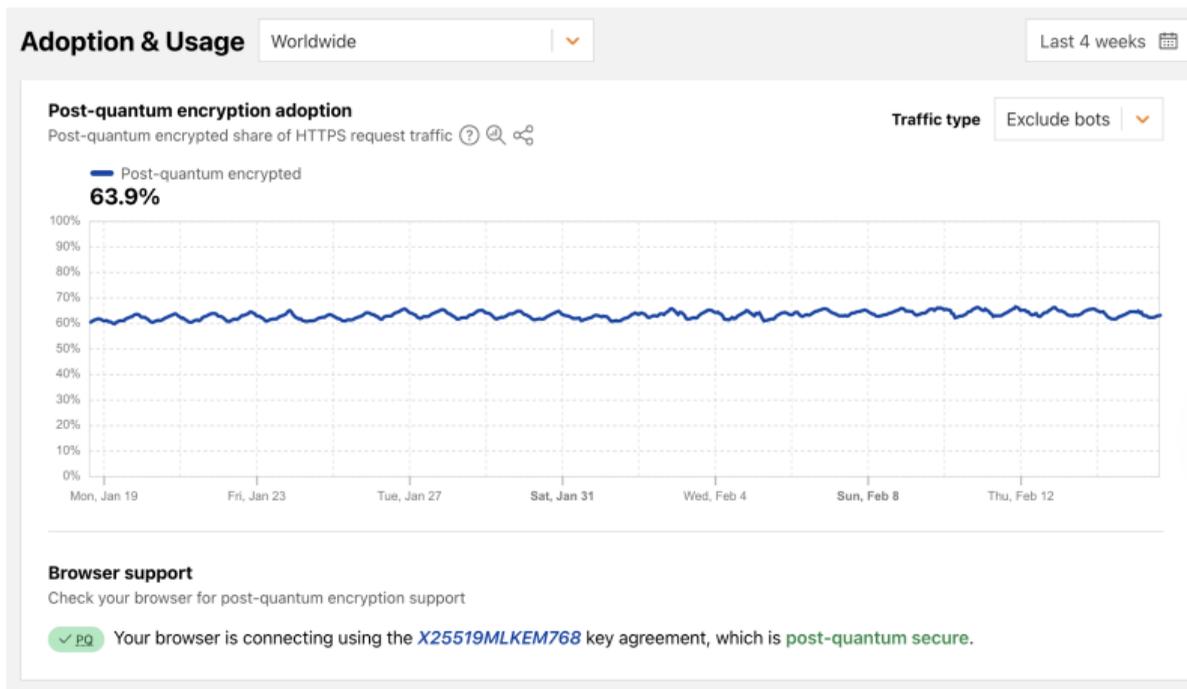


Figure: <https://radar.cloudflare.com/adoption-and-usage#post-quantum-encryption-adoption>

[//radar.cloudflare.com/adoption-and-usage#post-quantum-encryption-adoption](https://radar.cloudflare.com/adoption-and-usage#post-quantum-encryption-adoption)



Figure: <https://github.blog/engineering/platform-security/post-quantum-security-for-ssh-access-on-github/>



Quantum Resistance and the Signal Protocol

ehrenkret on 19 Sep 2023

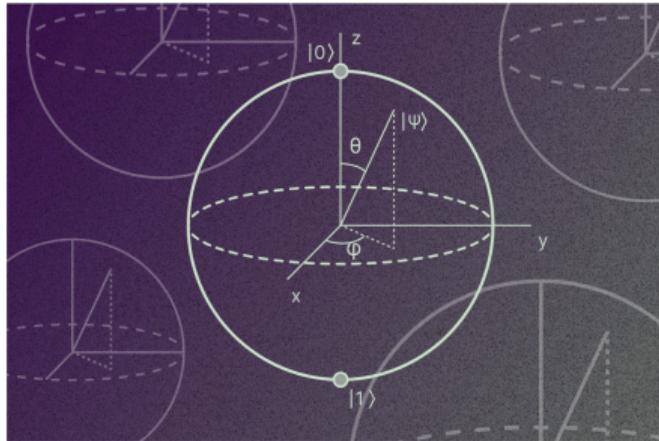


Figure: <https://signal.org/blog/pqxdh>

Quantum-Secure Cryptography in Messaging Apps

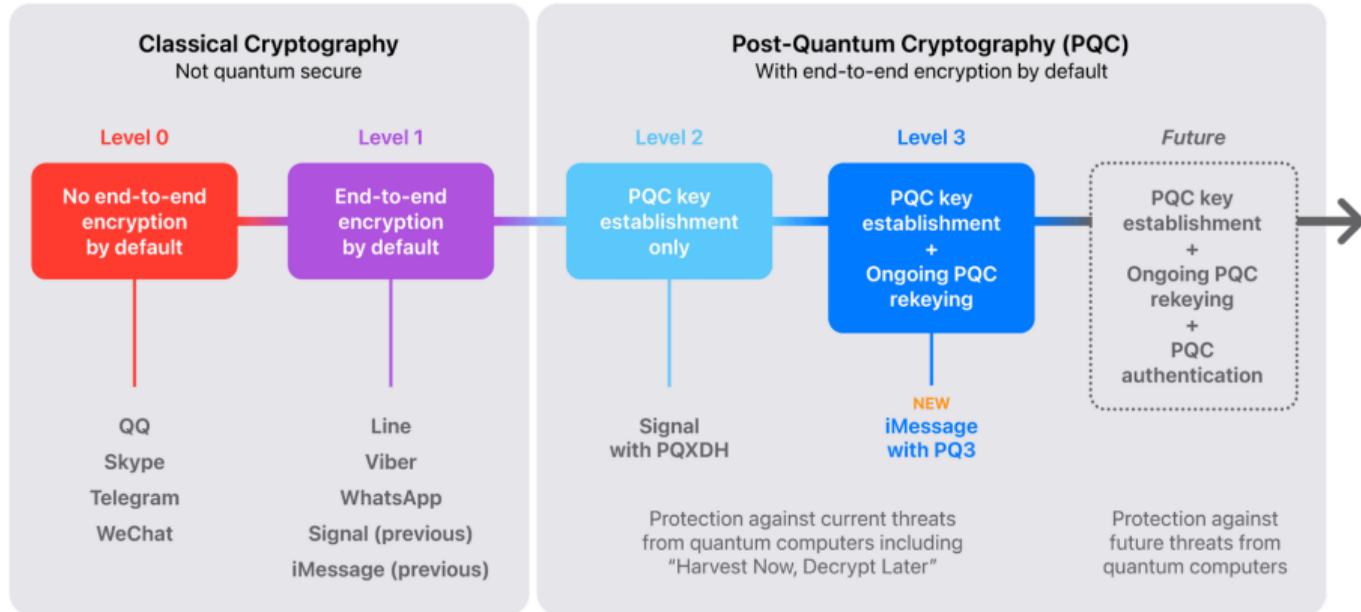


Figure: <https://security.apple.com/blog/imessage-pq3>

Crypto Categories

No Changes
Necessary

Symmetric Cryptography:

- AES
- SHA-256 / SHA-3
- HMAC
- etc.

Done.

Almost Drop-in
Replacements

NIST standardizations:

- Public Key Encryption
- Key Exchange
- Digital Signatures

A few other things:

- Identity-Based Encryption

Almost standards. Ready for
deployment.

Serious Alterations
of Protocols
Required

Advanced Primitives:

- Zero-Knowledge Proofs
- Distributed Privacy
- Many blockchain
privacy applications

Lots of recent progress on design. Near-
optimality has just been achieved for
certain primitives. Implementation
starting at ZRL.

Can Only Be Done
with Lattice
Cryptography

- Fully-Homomorphic Encryption (FHE) -
computation over
encrypted data
- Some Obfuscation (still
unclear if it can be
efficient or have any
useful applications)

Implementation /
deployment of
FHE at Haifa.

Contents

Quantum-Safe Cryptography

New Hardness Assumption

ML-KEM (CRYSTALS-Kyber)

ML-DSA (CRYSTALS-Dilithium)

Standards and Implementation

Today: Decisional Diffie-Hellman

Let \mathbb{G} be a group of prime order p and g be a generator for \mathbb{G} .

Sample a, b, c uniformly at random from \mathbb{Z}_p . The Decisional Diffie-Hellman problem is to distinguish the two cases:

$$(g, g^a, g^b, g^{ab})$$

$$(g, g^a, g^b, g^c)$$

New: Learning With Errors (LWE)

Definition 1. For positive integers m, n, q , and $\beta < q$, the $\text{LWE}_{n,m,q,\beta}$ problem asks to distinguish between the following two distributions:

1. $(\mathbf{A}, \mathbf{A}\mathbf{s} + \mathbf{e})$, where $\mathbf{A} \leftarrow \mathbb{Z}_q^{n \times m}$, $\mathbf{s} \leftarrow [\beta]^m$, $\mathbf{e} \leftarrow [\beta]^n$
2. (\mathbf{A}, \mathbf{u}) , where $\mathbf{A} \leftarrow \mathbb{Z}_q^{n \times m}$ and $\mathbf{u} \leftarrow \mathbb{Z}_q^n$.

LWE Hardness

The Learning With Errors problem gets harder when...

- ▶ the dimension gets larger
- ▶ the secret values gets larger
- ▶ the modulus gets smaller

Today: Computational Diffie-Hellman

Let \mathbb{G} be a group of prime order p and g be a generator for \mathbb{G} .

Sample a, b uniformly at random from \mathbb{Z}_p . The Computational Diffie-Hellman problem is, given g, g^a , and g^b , to find g^{ab} in \mathbb{G} .

New: Short Integer Solution (SIS)

Definition 4. For positive integers m, n, q , and $\beta < q$, the $\text{SIS}_{n,m,q,\beta}$ problem asks to find, for a randomly-chosen matrix $\mathbf{A} \leftarrow \mathbb{Z}_q^{n \times m}$, vectors $\mathbf{s}_1 \in [\beta]^m$ and $\mathbf{s}_2 \in [\beta]^n$ such that $\mathbf{A}\mathbf{s}_1 + \mathbf{s}_2 = \mathbf{0} \pmod{q}$.

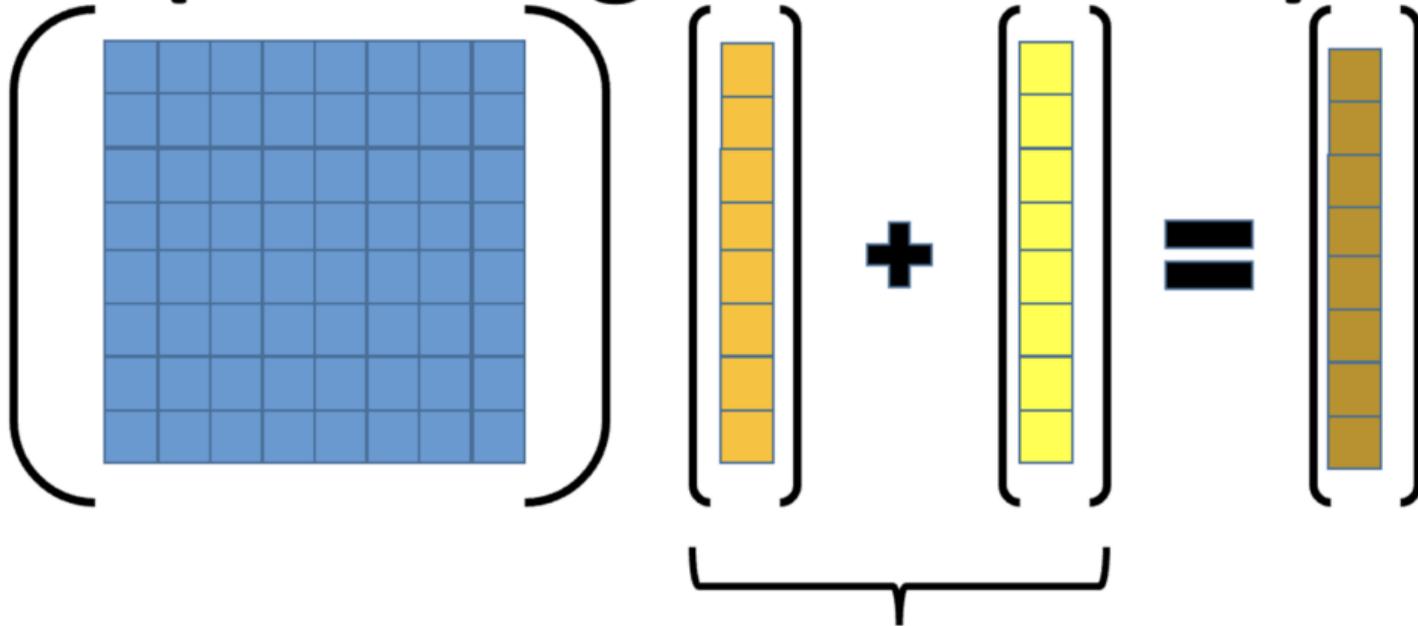
SIS Hardness

The Short Integer Solution problem gets harder when...

- ▶ the dimension gets larger
- ▶ the secret values gets smaller
- ▶ the modulus gets larger

This is opposite of LWE with respect to norms!

(Learning With Errors)



Small coefficients to enforce uniqueness

Hardness of LWE and SIS

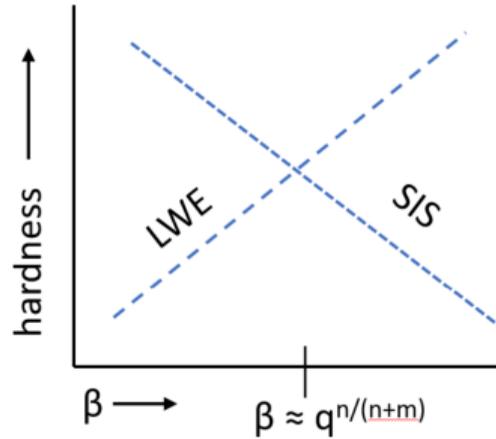


Figure 2: The hardness of $\text{LWE}_{n,m,q,\beta}$ and $\text{SIS}_{n,m,q,\beta}$ for fixed n, m, q , and varying β . The lines are not meant to describe the concrete hardness of these problems, but rather to illustrate the dependence of the hardness of these problems on β . The intersection point is approximately at $\beta = q^{n/(n+m)}$.

Basic Lattice Cryptography

The concepts behind Kyber (ML-KEM) and Dilithium (ML-DSA)

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(Last updated: June 18, 2025)

Figure: <https://eprint.iacr.org/2024/1287.pdf>

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Standards and Implementation

FIPS 203

Federal Information Processing Standards Publication

Module-Lattice-Based Key-Encapsulation Mechanism Standard

Category: Computer Security

Subcategory: Cryptography

Figure: <https://nvlpubs.nist.gov/nistpubs/fips/nist.fips.203.pdf>

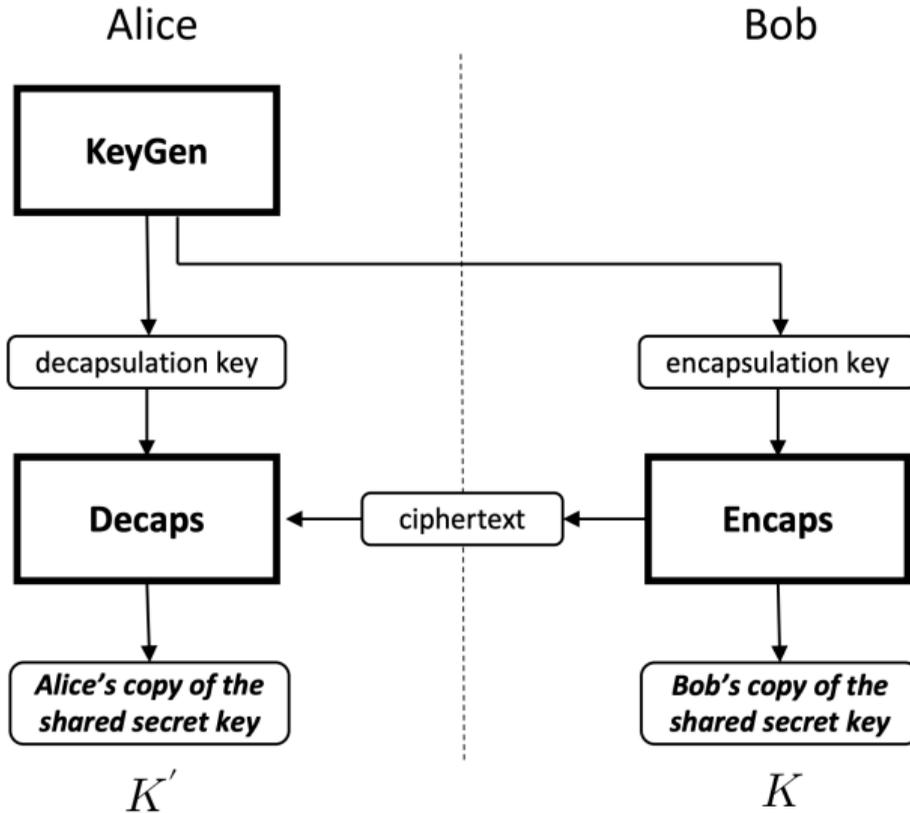
Defining a KEM

Definition 1 (Key Encapsulation Mechanism (KEM)). A key encapsulation mechanism is a triple of algorithms $\text{KEM} = \{\text{KeyGen}, \text{Enc}, \text{Dec}\}$ with public keyspace \mathcal{PK} , private keyspace \mathcal{SK} , ciphertext space \mathcal{C} , and shared keyspace \mathcal{K} . The triple of algorithms is defined as:

- $\text{KEM.KeyGen}() \text{ } \$ \rightarrow (sk, pk)$ Randomized algorithm that outputs a secret (private) key $sk \in \mathcal{SK}$, and a public key $pk \in \mathcal{PK}$.
- $\text{KEM.Enc}(pk) \text{ } \$ \rightarrow (k, c)$ Randomized algorithm that, given a public key $pk \in \mathcal{PK}$, outputs a shared key $k \in \mathcal{K}$, and a ciphertext $c \in \mathcal{C}$.
- $\text{KEM.Dec}(c, sk) \rightarrow y \in \{k, \perp\}$ Deterministic algorithm that, given a secret key, $sk \in \mathcal{SK}$ and a ciphertext $c \in \mathcal{C}$, returns the shared key $k \in \mathcal{K}$. In case of rejection, this algorithm returns \perp .

Figure: <https://cic.iacr.org/p/1/1/21/pdf>

KEM Overview



Today: ElGamal

Let \mathbb{G} be a group of prime order p and g be a generator for \mathbb{G} . Denote by p the public parameters (\mathbb{G}, g, p) .

Let the secret key $sk \leftarrow \$ \mathbb{Z}_p$ be sampled uniformly at random, and let the public key be $pk = g^{sk}$, where pk is made public.

The ElGamal encryption scheme, with message $m \in \mathbb{G}$, works as follows:

Enc : Sample uniform $x \leftarrow \$ \mathbb{Z}_p$ and encrypt as $X = g^x$ and $Y = pk^x \cdot m$.

Dec : Decrypt the ciphertext (X, Y) to get the message $m = Y \cdot X^{-sk}$.

ML-KEM KGen and Enc

$$\text{sk} : \mathbf{s} \leftarrow [\beta]^m, \text{pk} : (\mathbf{A} \leftarrow \mathbb{Z}_q^{m \times m}, \mathbf{t} = \mathbf{A}\mathbf{s} + \mathbf{e}_1), \text{ where } \mathbf{e}_1 \leftarrow [\beta]^m. \quad (6)$$

To encrypt a message $\mu \in \{0, 1\}$, the encryptor chooses $\mathbf{r}, \mathbf{e}_2 \leftarrow [\beta]^m$ and $e_3 \leftarrow [\beta]$, and outputs

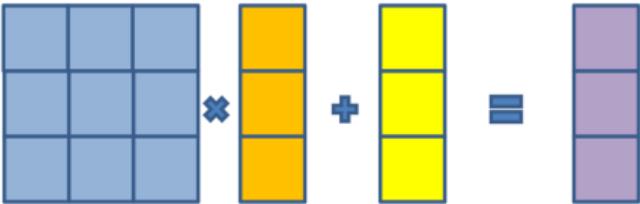
$$\left(\mathbf{u}^T = \mathbf{r}^T \mathbf{A} + \mathbf{e}_2^T, v = \mathbf{r}^T \mathbf{t} + e_3 + \left\lceil \frac{q}{2} \right\rceil \mu \right). \quad (7)$$

To decrypt, one computes $v - \mathbf{u}^T \hat{\mathbf{s}}$. But rather than this cleanly giving us the message μ as in (4), we instead obtain

$$v - \mathbf{u}^T \mathbf{s} = \mathbf{r}^T (\mathbf{A} \mathbf{s} + \mathbf{e}_1) + e_3 + \frac{q}{2} \mu - (\mathbf{r}^T \mathbf{A} + \mathbf{e}_2^T) \mathbf{s} \quad (8)$$

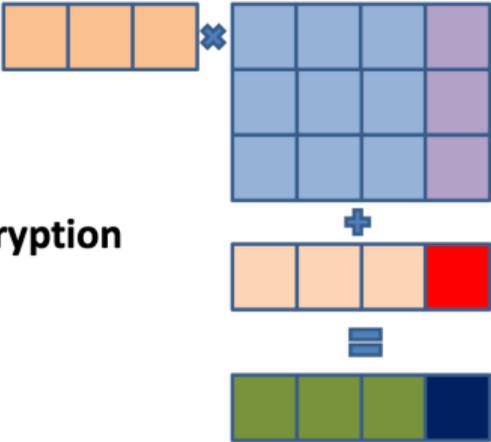
$$= \mathbf{r}^T \mathbf{e}_1 + e_3 + \frac{q}{2} \mu - \mathbf{e}_2^T \mathbf{s} \quad (9)$$

Visualization of ML-KEM

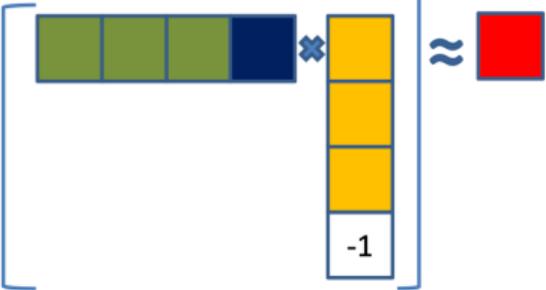


Public Key / Secret Key
Generation

Encryption



Decryption



Classical vs Quantum-Safe Key-Exchange

		Size keyshares(in bytes)		Ops/sec (higher is better)	
Algorithm	PQ	Client	Server	Client	Server
Kyber512	✓	800	768	50,000	100,000
Kyber768	✓	1,184	1,088	31,000	70,000
X25519	✗	32	32	17,000	17,000

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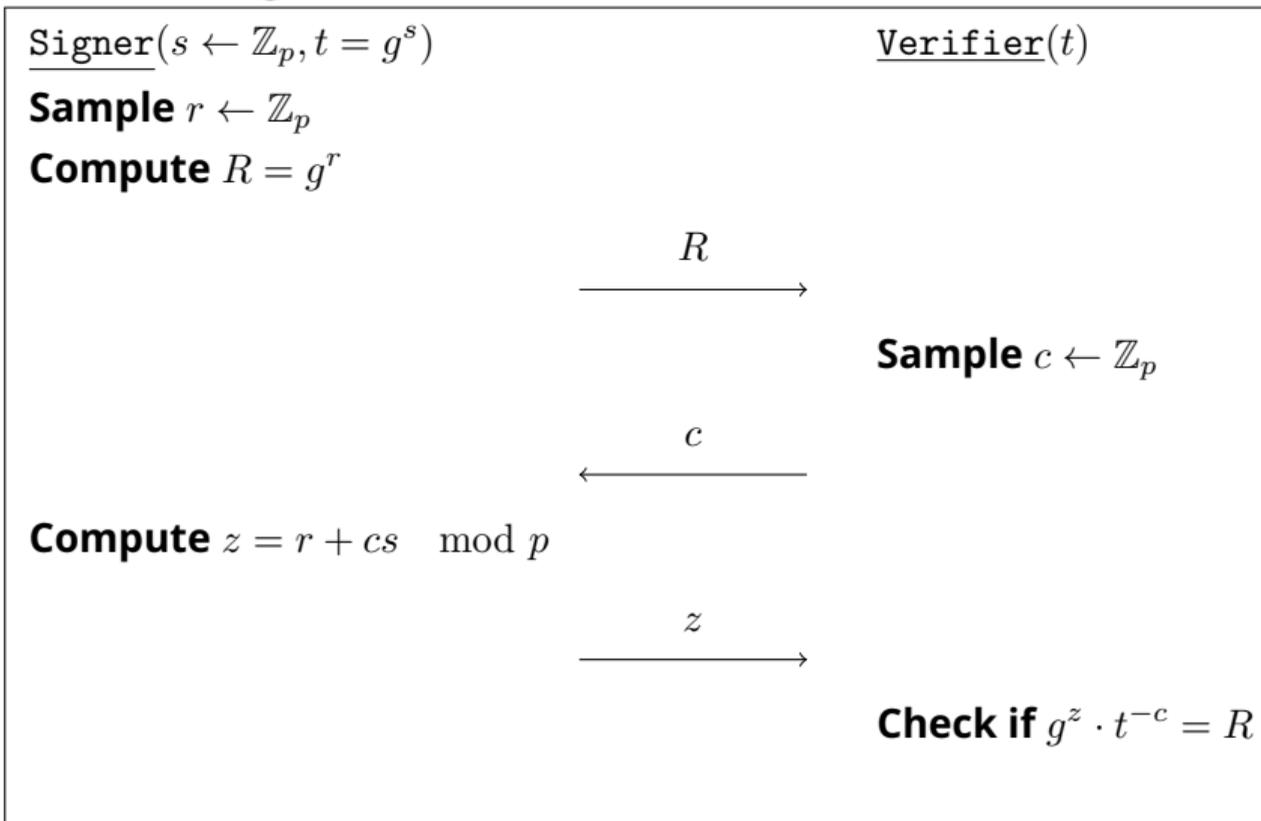
New Hardness Assumption

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Standards and Implementation

Today: Schnorr Signatures (interactive)



ML-DSA (interactive)

Private information: $\mathbf{s}_1 \in [\beta]^m, \mathbf{s}_2 \in [\beta]^n$

Public information: $\mathbf{A} \in \mathcal{R}_{q,f}^{n \times m}, \mathbf{t} = \mathbf{A}\mathbf{s}_1 + \mathbf{s}_2 \in \mathcal{R}_{q,f}^n$

Prover

$$\mathbf{y}_1 \leftarrow [\gamma + \bar{\beta}]^m$$

$$\mathbf{y}_2 \leftarrow [\gamma + \bar{\beta}]^n,$$

$$\mathbf{w} := \mathbf{A}\mathbf{y}_1 + \mathbf{y}_2$$

$$\mathbf{z}_1 := \mathbf{c}\mathbf{s}_1 + \mathbf{y}_1$$

$$\mathbf{z}_2 := \mathbf{c}\mathbf{s}_2 + \mathbf{y}_2$$

if $\mathbf{z}_1 \notin [\bar{\beta}]^m$ or $\mathbf{z}_2 \notin [\bar{\beta}]^n$

then $(\mathbf{z}_1, \mathbf{z}_2) := \perp$

Verifier

$$\xrightarrow{\mathbf{w}}$$

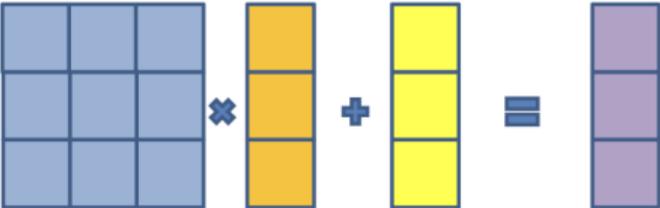
$$c \leftarrow \mathcal{C}$$

$$\xleftarrow{c}$$

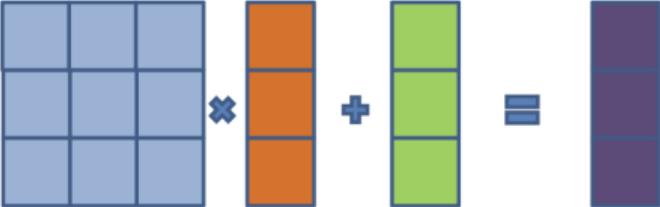
$$\xrightarrow{(\mathbf{z}_1, \mathbf{z}_2)}$$

Accept iff $\mathbf{z}_1 \in [\bar{\beta}]^m$ and $\mathbf{z}_2 \in [\bar{\beta}]^n$
and $\mathbf{A}\mathbf{z}_1 + \mathbf{z}_2 - \mathbf{c}\mathbf{t} = \mathbf{w}$

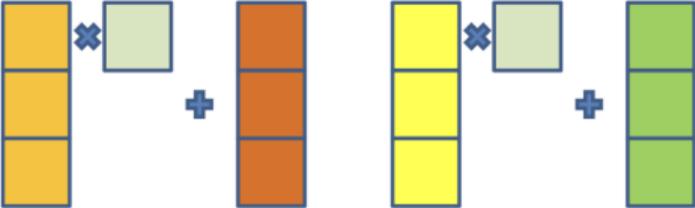
Visualization of ML-DSA



Public Key / Secret Key
Generation



$$\square = H(\begin{bmatrix} \square \\ \square \\ \square \end{bmatrix}, \mu)$$



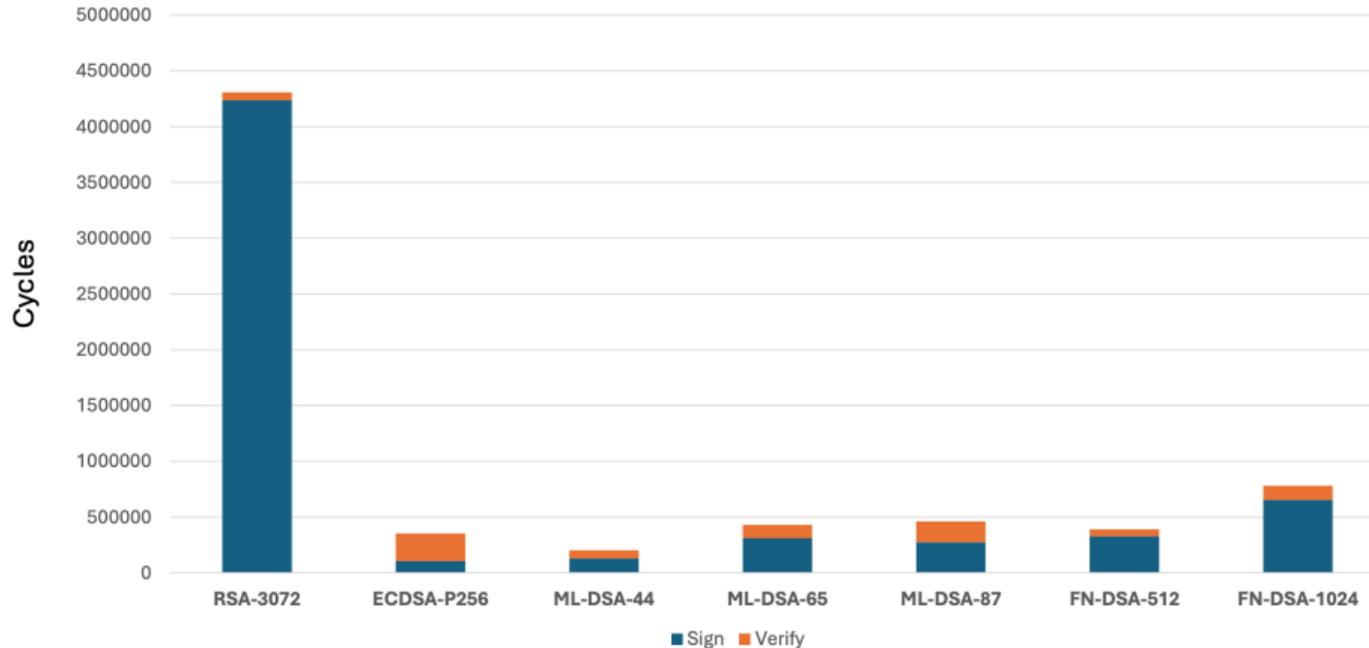
Schnorr vs ML-DSA

- ▶ Schnorr is based on DLOG, ML-DSA on LWE and SIS
- ▶ Schnorr has uniform challenges, ML-DSA has small
- ▶ ML-DSA aborts if the norm of z is too big
- ▶ ML-DSA require norm checks of the signature

PQC Key and Signature Sizes

Scheme	Public Key (bytes)	Private Key (bytes)	Signature (bytes)	Security Level
RSA-3072	384	384	384	Classical-128
ECDSA-P256	64	32	256	Classical-128
ML-DSA-44 (Dilithium2)	1312	2528	2420	PQC Category 2 (SHA3-256)
ML-DSA-65 (Dilithium3)	1952	4000	3293	PQC Category 3 (AES-192)
ML-DSA-87 (Dilithium5)	2592	4864	4595	PQC Category 5 (AES-256)
FN-DSA-512 (Falcon512)	897	7553	666	PQC Category 1 (AES-128)
FN-DSA-1024 (Falcon1024)	1793	13953	1280	PQC Category 5 (AES-256)

PQC Signatures– Performance

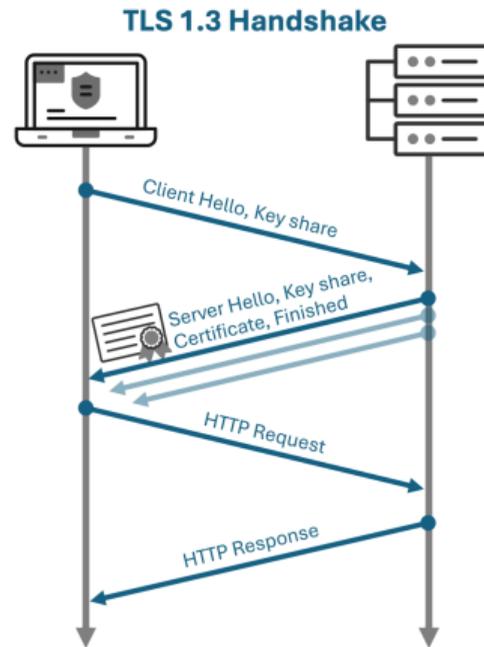


Signatures in TLS

A bit much to chew?



- TLS & WebPKI Certificate Signatures
 - *Server Certificate*: 1 public key and signature, 2 SCT signatures
 - *Intermediate CA Certificate*: 1 public key and signature
 - *TLS Handshake*: 1 signature
 - ML-DSA-44 → **14,724 bytes**
 - Current Quantum-Vulnerable → **1,248 bytes**
- ML-KEM-768 key shares
 - Client → Server: 1,184 bytes
 - Server → Client: **1,088 bytes**
- Why does this matter?
 - *TCP initial congestion window* limits the first wave of messages
 - Typical default: **~14,600 bytes**
- Without protocol/implementation changes, this could slow web connection establishment



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Notation from ML-KEM

n	Denotes the integer 256 throughout this document.
q	Denotes the prime integer $3329 = 2^8 \cdot 13 + 1$ throughout this document.
ζ	Denotes the integer 17, which is a primitive n -th root of unity modulo q .
\mathbb{B}	The set $\{0, 1, \dots, 255\}$ of unsigned 8-bit integers (bytes).
\mathbb{Q}	The set of rational numbers.
\mathbb{Z}	The set of integers.
\mathbb{Z}_m	The ring of integers modulo m (i.e., the set $\{0, 1, \dots, m - 1\}$ equipped with the operations of addition and multiplication modulo m .)
\mathbb{Z}_m^n	The set of n -tuples over \mathbb{Z}_m equipped with \mathbb{Z}_m -module structure. As a data type, this is the set of length- n arrays whose entries are in \mathbb{Z}_m .
R_q	The ring $\mathbb{Z}_q[X]/(X^n + 1)$ consisting of polynomials of the form $f = f_0 + f_1X + \dots + f_{255}X^{255}$, where $f_j \in \mathbb{Z}_q$ for all j . The ring operations are addition and multiplication modulo $X^n + 1$.

ML-KEM Key Generation

Algorithm 13 K-PKE.KeyGen(d)

Uses randomness to generate an encryption key and a corresponding decryption key.

Input: randomness $d \in \mathbb{B}^{32}$.

Output: encryption key $ek_{\text{PKE}} \in \mathbb{B}^{384k+32}$.

Output: decryption key $dk_{\text{PKE}} \in \mathbb{B}^{384k}$.

```
1:  $(\rho, \sigma) \leftarrow G(d\|k)$  ▷ expand 32+1 bytes to two pseudorandom 32-byte seeds1
2:  $N \leftarrow 0$ 
3: for  $(i \leftarrow 0; i < k; i++)$  ▷ generate matrix  $\hat{\mathbf{A}} \in (\mathbb{Z}_q^{256})^{k \times k}$ 
4:   for  $(j \leftarrow 0; j < k; j++)$ 
5:      $\hat{\mathbf{A}}[i, j] \leftarrow \text{SampleNTT}(\rho\|j\|i)$  ▷  $j$  and  $i$  are bytes 33 and 34 of the input
6:   end for
7: end for
8: for  $(i \leftarrow 0; i < k; i++)$  ▷ generate  $\mathbf{s} \in (\mathbb{Z}_q^{256})^k$ 
9:    $\mathbf{s}[i] \leftarrow \text{SamplePolyCBD}_{\eta_1}(\text{PRF}_{\eta_1}(\sigma, N))$  ▷  $\mathbf{s}[i] \in \mathbb{Z}_q^{256}$  sampled from CBD
10:   $N \leftarrow N + 1$ 
11: end for
12: for  $(i \leftarrow 0; i < k; i++)$  ▷ generate  $\mathbf{e} \in (\mathbb{Z}_q^{256})^k$ 
13:    $\mathbf{e}[i] \leftarrow \text{SamplePolyCBD}_{\eta_1}(\text{PRF}_{\eta_1}(\sigma, N))$  ▷  $\mathbf{e}[i] \in \mathbb{Z}_q^{256}$  sampled from CBD
14:   $N \leftarrow N + 1$ 
15: end for
16:  $\hat{\mathbf{s}} \leftarrow \text{NTT}(\mathbf{s})$  ▷ run NTT  $k$  times (once for each coordinate of  $\mathbf{s}$ )
17:  $\hat{\mathbf{e}} \leftarrow \text{NTT}(\mathbf{e})$  ▷ run NTT  $k$  times
18:  $\hat{\mathbf{t}} \leftarrow \hat{\mathbf{A}} \circ \hat{\mathbf{s}} + \hat{\mathbf{e}}$  ▷ noisy linear system in NTT domain
19:  $ek_{\text{PKE}} \leftarrow \text{ByteEncode}_{12}(\hat{\mathbf{t}})\|\rho$  ▷ run  $\text{ByteEncode}_{12}$   $k$  times, then append  $\hat{\mathbf{A}}$ -seed
20:  $dk_{\text{PKE}} \leftarrow \text{ByteEncode}_{12}(\hat{\mathbf{s}})$  ▷ run  $\text{ByteEncode}_{12}$   $k$  times
21: return  $(ek_{\text{PKE}}, dk_{\text{PKE}})$ 
```

ML-DSA Sign Part 1

Algorithm 7 $\text{ML-DSA.Sign_internal}(sk, M', rnd)$

Deterministic algorithm to generate a signature for a formatted message M' .

Input: Private key $sk \in \mathbb{B}^{32+32+64+32 \cdot ((\ell+k) \cdot \text{bitlen}(2\eta)+dk)}$, formatted message $M' \in \{0, 1\}^*$, and per message randomness or dummy variable $rnd \in \mathbb{B}^{32}$.

Output: Signature $\sigma \in \mathbb{B}^{\lambda/4 + \ell \cdot 32 \cdot (1 + \text{bitlen}(\gamma_1 - 1)) + \omega + k}$.

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1:  $(\rho, K, tr, \mathbf{s}_1, \mathbf{s}_2, \mathbf{t}_0) \leftarrow \text{skDecode}(sk)$ 
2:  $\hat{\mathbf{s}}_1 \leftarrow \text{NTT}(\mathbf{s}_1)$ 
3:  $\hat{\mathbf{s}}_2 \leftarrow \text{NTT}(\mathbf{s}_2)$ 
4:  $\hat{\mathbf{t}}_0 \leftarrow \text{NTT}(\mathbf{t}_0)$ 
5:  $\hat{\mathbf{A}} \leftarrow \text{ExpandA}(\rho)$   $\triangleright \mathbf{A}$  is generated and stored in NTT representation as  $\hat{\mathbf{A}}$ 
6:  $\mu \leftarrow \text{H}(\text{BytesToBits}(tr) || M', 64)$   $\triangleright$  message representative that may optionally be
   computed in a different cryptographic module
7:  $\rho'' \leftarrow \text{H}(K || rnd || \mu, 64)$   $\triangleright$  compute private random seed
8:  $\kappa \leftarrow 0$   $\triangleright$  initialize counter  $\kappa$ 
9:  $(\mathbf{z}, \mathbf{h}) \leftarrow \perp$ 
10: while  $(\mathbf{z}, \mathbf{h}) = \perp$  do  $\triangleright$  rejection sampling loop
11:    $\mathbf{y} \in R_q^\ell \leftarrow \text{ExpandMask}(\rho'', \kappa)$ 
12:    $\mathbf{w} \leftarrow \text{NTT}^{-1}(\hat{\mathbf{A}} \circ \text{NTT}(\mathbf{y}))$ 
13:    $\mathbf{w}_1 \leftarrow \text{HighBits}(\mathbf{w})$   $\triangleright$  signer's commitment
14:    $\triangleright$  HighBits is applied componentwise (see explanatory text in Section 7.4)
15:    $\tilde{c} \leftarrow \text{H}(\mu || \text{w1Encode}(\mathbf{w}_1), \lambda/4)$   $\triangleright$  commitment hash
16:    $c \in R_q \leftarrow \text{SampleInBall}(\tilde{c})$   $\triangleright$  verifier's challenge
17:    $\hat{c} \leftarrow \text{NTT}(c)$ 
18:    $\langle\langle cs_1 \rangle\rangle \leftarrow \text{NTT}^{-1}(\hat{c} \circ \hat{\mathbf{s}}_1)$ 
19:    $\langle\langle cs_2 \rangle\rangle \leftarrow \text{NTT}^{-1}(\hat{c} \circ \hat{\mathbf{s}}_2)$ 
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ML-DSA Sign Part 2

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20:  $\mathbf{z} \leftarrow \mathbf{y} + \langle\langle \mathbf{cs}_1 \rangle\rangle$  ▷ signer's response
21:  $\mathbf{r}_0 \leftarrow \text{LowBits}(\mathbf{w} - \langle\langle \mathbf{cs}_2 \rangle\rangle)$ 
22: ▷ LowBits is applied componentwise (see explanatory text in Section 7.4)
23: if  $\|\mathbf{z}\|_\infty \geq \gamma_1 - \beta$  or  $\|\mathbf{r}_0\|_\infty \geq \gamma_2 - \beta$  then  $(\mathbf{z}, \mathbf{h}) \leftarrow \perp$  ▷ validity checks
24: else
25:    $\langle\langle \mathbf{ct}_0 \rangle\rangle \leftarrow \text{NTT}^{-1}(\hat{c} \circ \hat{\mathbf{t}}_0)$ 
26:    $\mathbf{h} \leftarrow \text{MakeHint}(-\langle\langle \mathbf{ct}_0 \rangle\rangle, \mathbf{w} - \langle\langle \mathbf{cs}_2 \rangle\rangle + \langle\langle \mathbf{ct}_0 \rangle\rangle)$  ▷ Signer's hint
27:   ▷ MakeHint is applied componentwise (see explanatory text in Section 7.4)
28:   if  $\|\langle\langle \mathbf{ct}_0 \rangle\rangle\|_\infty \geq \gamma_2$  or the number of 1's in  $\mathbf{h}$  is greater than  $\omega$ , then  $(\mathbf{z}, \mathbf{h}) \leftarrow \perp$ 
29:   end if
30: end if
31:  $\kappa \leftarrow \kappa + \ell$  ▷ increment counter
32: end while
33:  $\sigma \leftarrow \text{sigEncode}(\tilde{c}, \mathbf{z} \bmod^\pm q, \mathbf{h})$ 
34: return  $\sigma$ 
```

Implementations

- ▶ Reference implementations in C/AVX2 at <https://pq-crystals.org>
- ▶ Educational implementations in Python:
 - ▶ <https://github.com/GiacomoPope/kyber-py>
 - ▶ <https://github.com/GiacomoPope/dilithium-py>
- ▶ Open Quantum Safe project: <https://openquantumsafe.org>

Questions?