



NTNU

Norwegian University of
Science and Technology

PADDING ORACLES: RSA ENCRYPTION

TTM4205 – Lecture 12

Tjerand Silde

18.10.2024

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Announcements

Padding Oracles

Recall: RSA Encryption

RSA Padding Schemes

The Bleichenbacher Attack

Improved Bleichenbacher Attack

RSA Padding Oracle Mitigations

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Reference Group Meeting

We have four reference group members:

- ▶ Adrian Tokle Storset (adriats), from MSTCNNS
- ▶ Daniel Nils Braun (daniembr), exchange student
- ▶ Jiaqi Chen (jiaqic), from SECCLLO
- ▶ Emil Bragstad (emil.bragstad), from MTKOM

The next meeting will be on October 21st. Please provide feedback!

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Reference Material

These slides are based on:

- ▶ The referred papers in the slides
- ▶ JPA: parts of chapter 10
- ▶ DW: parts of chapter 6

Padding Oracles

By this we mean, on a high level, an API that allows an adversary to learn if some input is correctly formed.

We limit ourselves to inputs with a particular padding.

Padding Oracles

We will look at symmetric and asymmetric padding schemes:

- ▶ in depth on the CBC block cipher mode (last time)
- ▶ extension attack against hashing (last time)
- ▶ padding attacks against the RSA scheme (today)

Several of which are relevant to the weekly problems.

We will also look at some mitigations to these issues.

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Textbook RSA

The plain RSA encryption scheme works as follows:

KGen:

- ▶ Samples primes p and q of appropriate size and entropy
- ▶ Use fixed e and compute $d \equiv e^{-1} \pmod{\text{lcm}(p-1, q-1)}$
- ▶ Output the key pair $\text{pk} = (e, n = p \cdot q)$ and $\text{sk} = (d, p, q)$

Textbook RSA

The plain RSA encryption scheme works as follows:

Enc:

- ▶ Takes as input a message m and public key $pk = (e, n)$
- ▶ Computes the ciphertext $c \equiv m^e \pmod n$ and outputs c

Textbook RSA

The plain RSA encryption scheme works as follows:

Dec:

- ▶ Takes as input a ciphertext c and secret key $sk = (d, p, q)$
- ▶ Computes the message $m \equiv c^d \pmod{p \cdot q}$ and outputs m

Textbook RSA

Question: Why is not the textbook RSA scheme secure?

Textbook RSA

The following things make the RSA scheme insecure:

- ▶ It is not randomized and hence not even CPA secure
- ▶ Given a ciphertext you can search for the message
- ▶ High-entropy messages still gives the same ciphertext
- ▶ (The Jacobi symbol of m and c will be the same)

Solution: structured, but randomized padding

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RSA-PKCS#1v1.5

Let n be of k bytes. Given a message m of $\ell \leq k - 11$ bytes, the padded messages \bar{m} of length k bytes is constructed as follows:

00 02 {at least 8 non-zero random bytes} 00 { m }

Quite simple, not proven secure, not secure in practice...

A bad couple of years for the cryptographic token industry



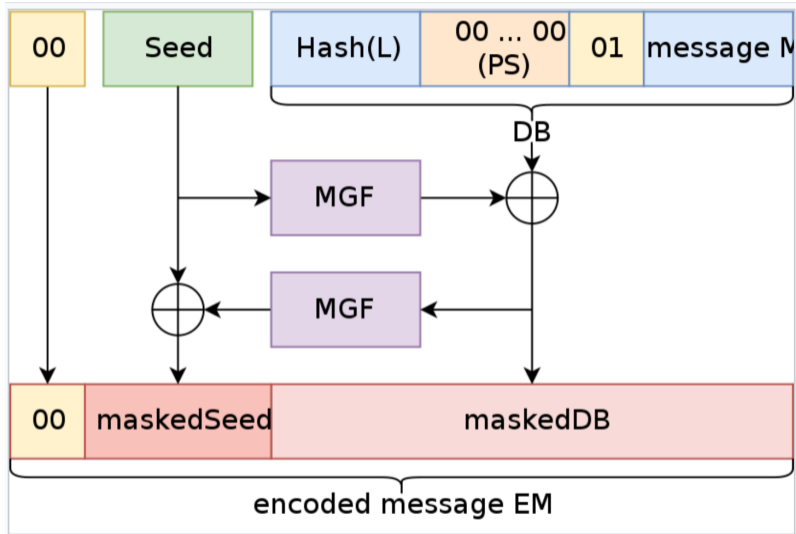
SafeNet eToken PRO Anywhere

Figure: <https://blog.cryptographyengineering.com/2012/06/21/bad-couple-of-years-for-cryptographic>

More complex, proven secure, what you should use:

- ▶ Let n be of k bytes and message m be of ℓ bytes.
- ▶ Let MGF and Hash be hash functions with output h bytes.
- ▶ Let L be a label (which can be set to the all zero string)
- ▶ Let seed be an ephemeral random string of h bytes.
- ▶ Let PS be a all zero string of length $k - \ell - 2h - 2$ bytes.

OAEP



Optimal Asymmetric Encryption — How to Encrypt with RSA

MIHIR BELLARE*

PHILLIP ROGAWAY†

November 19, 1995

Figure: <https://cseweb.ucsd.edu/~mihir/papers/oaep.pdf>

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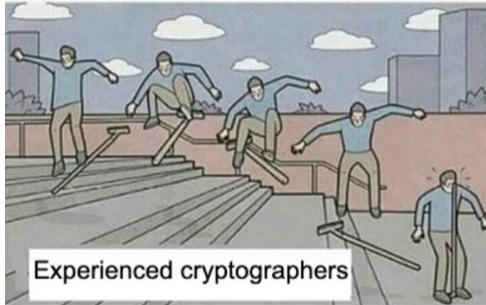
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New cryptographers



Experienced cryptographers

The Bleichenbacher Attack

However, many implementations (still) use RSA-PKCS#1v1.5 or similar padding schemes (note that this is version 1.5).

Recall: $00\ 02\ \{\text{at least 8 non-zero random bytes}\}\ 00\ \{m\}$.

Question: Assuming no integrity check of RSA ciphertexts, how could you attack this scheme?

The Bleichenbacher Attack

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- ▶ We know that if valid then $2 \cdot 2^{8(k-2)} \leq \bar{m} \cdot r < 3 \cdot 2^{8(k-2)}$.
- ▶ Repeat for fresh values r until we have a unique \bar{m} left.

Chosen Ciphertext Attacks Against Protocols Based on the RSA Encryption Standard PKCS #1

Daniel Bleichenbacher

Bell Laboratories
700 Mountain Ave.
Murray Hill, NJ 07974
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Figure: <https://spar.isi.jhu.edu/~mgreen/bleichenbacher.pdf>

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Improving the Attack

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3. Parallelization and threading and pre-computation
4. Adapt based on how strict padding checks are

Padding Checks

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3. FTT: same as above, but also allows 0s in the "non-zero random bytes".
4. TFT: same as above, but 'ok' even if there are no zeros after the first byte.
5. TTT: padding is 'ok' as long as it starts with 0x 00 02.



Oracle	Original algorithm		Modified algorithm			
	Mean	Median	Mean	Median	Trimmers	Mean skipped
FFF	-	-	18 040 221	12 525 835	50 000	7 321
FFT	215 982	163 183	49 001	14 501	1 500	65 944
FTT	159 334	111 984	39 649	11 276	2 000	61 552
TFT	39 536	24 926	10 295	4 014	600	20 192
TTT	38 625	22 641	9 374	3 768	500	18 467

Table 1: Performance of the original and modified algorithms.

Figure: <https://eprint.iacr.org/2012/417.pdf>

Efficient Padding Oracle Attacks on Cryptographic Hardware*

Romain Bardou¹, Riccardo Focardi^{2**}, Yusuke Kawamoto^{3***},
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¹ INRIA SecSI, LSV, CNRS & ENS-Cachan, France

² DAIS, Università Ca' Foscari, Venezia, Italy

³ School of Computer Science, University of Birmingham, UK

⁴ INRIA Project ProSecCo, Paris, France

⁵ Department of Telematics, NTNU, Norway

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Protection

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- ▶ Encrypt-then-Authenticate

Protection

- ▶ Use OAEP padding for encryption
- ▶ Encrypt-then-Authenticate
- ▶ Do not use RSA for encryption

Questions?