NTNU | Norwegian University of Science and Technology

LEGACY CRYPTO 2

TTM4205 – Lecture 6

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07.09.2023

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Reference Group

I am looking for (at least) three students to form a reference group in this course, preferably students from different programs. We will meet three times during the semester, and your feedback is extremely valuable.

Send me an email and/or talk to me in the break :)

Deadlines

- Topic/scope/group approval: **November 1st**
- Short oral presentations: **November 23rd** (TBC)
- Draft submission for feedback: **November 23rd**
- "Weekly Problems": December 1st at 23:59.
- "Special Topics Project": December 22nd at 23:59

Contents

Legacy Crypto is...

- Old and outdated crypto
- Insecure, weakened, or flawed crypto
- Crypto regulated by export control
- Potentially backdoored crypto
- Key escrow and surveillance
- Downgradable crypto protocols

Two Categories

Secret Key Crypto

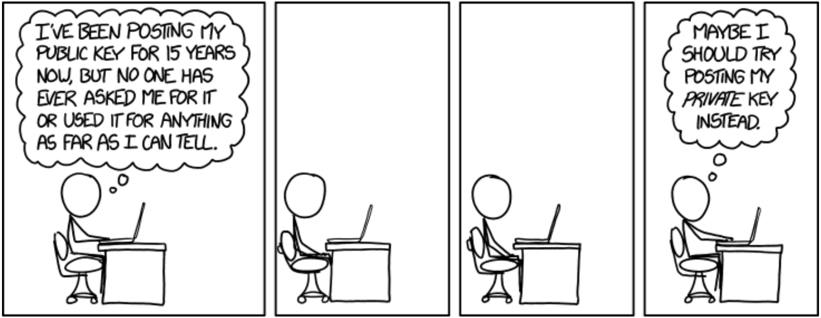
Public Key Crypto

Today

Secret Key Crypto

Public Key Crypto

Public Key Crypto



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Legacy Ciphers

While we have many attacks against symmetric key ciphers that made them obsolete, we do not have groundbreaking attacks against the legacy public key ciphers.

However, we need to be careful when setting parameters, and (as with CBC) be careful when using padding schemes.

Here are some examples…

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- Supersingular curves $\rightarrow$ Can break Decisional DH
- Choose safe curves? $\rightarrow$ Standardized P-256, X25519, ...

ECC in Practice

Elliptic Curve Cryptography in Practice

Joppe W. Bos[1], J. Alex Halderman[2], Nadia Heninger[3], Jonathan Moore, Michael Naehrig[1], and Eric Wustrow[2]

[1] Microsoft Research
[2] University of Michigan
[3] University of Pennsylvania

Figure: https://eprint.iacr.org/2013/734.pdf

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- PKCS 1 padding $\rightarrow$ Bleichenbacher's padding attack

RSA

Twenty Years of Attacks on the RSA Cryptosystem

Dan Boneh
dabo@cs.stanford.edu

Figure:
https://crypto.stanford.edu/~dabo/papers/RSA-survey.pdf

RSA Challenges

Challenge Name	Digits	Bits	Date Factored	Factored by
RSA-100	100	330	Apr 1, 1991	A. K. Lenstra
RSA-110	110	364	Apr 14, 1992	A. K. Lenstra and M.S. Manasse
RSA-120	120	397	Jul 9, 1993	T. Denny et al.
RSA-130	130	430	Apr 10, 1996	A. K. Lenstra et al.
RSA-140	140	463	Feb 2, 1999	H. te Riele et al.
RSA-150	150	496	Apr 16, 2004	K. Aoki et al.
RSA-155	155	512	Aug 22, 1999	H. te Riele et al.
RSA-160	160	530	Apr 1, 2003	J. Franke et al.
RSA-170	170	563	Dec 29, 2009	D. Bonenberger and M. Krone
RSA-576	174	576	Dec 3, 2003	J. Franke et al.
RSA-180	180	596	May 8, 2010	S. A. Danilov and I. A. Popovyan
RSA-190	190	629	Nov 8, 2010	A. Timofeev and I. A. Popovyan
RSA-640	193	640	Nov 2, 2005	J. Franke et al.
RSA-200	200	663	May 9, 2005	J. Franke et al.
RSA-210	210	696	Sep 26, 2013	R. Propper
RSA-704	212	704	Jul 2, 2012	S. Bai, E. Thomé and P. Zimmermann
RSA-220	220	729	May 13, 2016	S. Bai, P. Gaudry, A. Kruppa, E. Thomé and P. Zimmermann
RSA-230	230	762	Aug 15, 2018	S. S. Gross
RSA-768	232	768	Dec 12, 2009	T. Kleinjung et al.
RSA-240	240	795	Nov 24, 2019	F. Boudot, P. Gaudry, A. Guillevic, N. Heninger, E. Thomé and P. Zimmermann
RSA-250	250	829	Feb 28, 2020	F. Boudot, P. Gaudry, A. Guillevic, N. Heninger, E. Thomé and P. Zimmermann

Table 1. The solved RSA Challenges

Figure: https://eprint.iacr.org/2021/894.pdf

Key Sizes

	Parameter	Legacy	Future System Use Near Term	Future System Use Long Term
Symmetric Key Size	k	80	128	256
Hash Function Output Size	m	160	256	512
MAC Output Size	m	80	128	256
RSA Problem	$\ell(n) \geq$	1024	3072	15360
Finite Field DLP	$\ell(p^n) \geq$	1024	3072	15360
	$\ell(p), \ell(q) \geq$	160	256	512
ECDLP	$\ell(q) \geq$	160	256	512
Pairing	$\ell(q^n) \geq$	1024	3072	15360
	$\ell(p), \ell(q) \geq$	160	256	512

Table 2. Key Size Analysis, where $\ell(\cdot)$ refers to the bit-length of the parameter.

Figure: `https://eprint.iacr.org/2021/894.pdf`

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Static Finite Field DH

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- Export Diffie-Hellman accept 512 bit prime groups
- One week of pre-computation $\rightarrow$ DL takes 1 min
- 2015: two 1024 groups break 18% HTTPS and 26% SSH

Logjam Attack

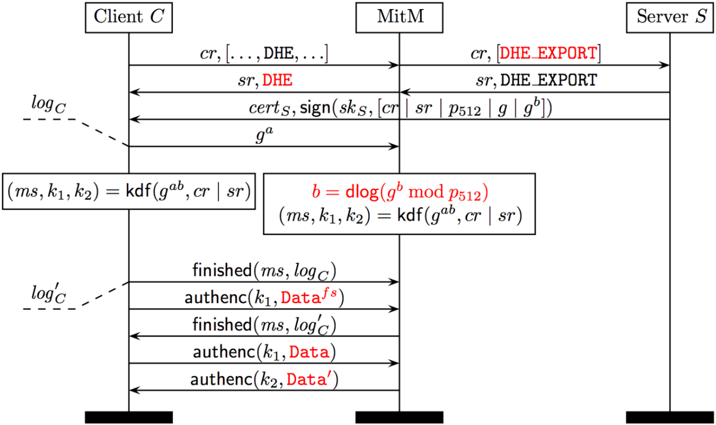

Figure: https://weakdh.org/imperfect-forward-secrecy.pdf

Old Attacks on TLS

RC4
- Roos's Bias 1995
- Fluhrer, Martin & Shamir 2001
- Klein 2005
- Combinatorial Problem 2001
- Royal Holloway 2013
- Bar-mitzvah 2015
- NOMORE 2015

RSA-PKCS#1 v1.5 Encryption
- Bleichenbacher 1998
- Jager 2015
- DROWN 2016

Renegotiation
- Marsh Ray Attack 2009
- Renegotiation DoS 2011
- Triple Handshake 2014

3DES
- Sweet32

AES-CBC
- Vaudenay 2002
- Boneh/Brumley 2003
- BEAST 2011
- Lucky13 2013
- POODLE 2014
- Lucky Microseconds 2015

Compression
- CRIME 2012

MD5 & SHA1
- SLOTH 2016
- SHAttered 2017

Figure: `https://owasp.org/www-chapter-london/assets/slides/OWASPLondon20180125_TLSv1.3_Andy_Brodie.pdf`

Downgrade Attacks on TLS

TLS: a long year of downgrade attacks

• POODLE	TLS 1.2 → SSLv3	[Dec'14]
• FREAK	RSA-2048 → RSA-512	[Mar'15]
• LOGJAM	DH-2048 → DH-512	[May'15]
• BLEICH?	RSA-Sign → RSA-Enc	[Aug'15]
• SLOTH	RSA-SHA256 → RSA-MD5	[Jan'16]

Figure: `https://rwc.iacr.org/2016/Slides/Downgrade.pdf`

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- Removed RSA for key exchange
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- Removed MD5 and SHA-1 hash functions
- Removed AES-CBC encryption mode
- Removed static (EC) Diffie-Hellman
- Only standardized groups/curves

New Cipher Suits

TLS 1.3 only allows for 5 different cipher suits:

- (EC)DHE-AES-128-GCM-SHA256
- (EC)DHE-AES-256GCM-SHA384
- (EC)DHE-CHACHA20-POLY1305-SHA256
- (EC)DHE-AES-128-CCM-SHA256
- (EC)DHE-AES-128-CCM-8-SHA256

Matthew Green's Blog

- Standards: https://blog.cryptographyengineering.com/2011/10/04/how-standards-go-wrong-constructive
- Logjam: https://blog.cryptographyengineering.com/2015/05/22/attack-of-week-logjam
- FREAK: https://blog.cryptographyengineering.com/2015/03/03/attack-of-week-freak-or-factoring-nsa

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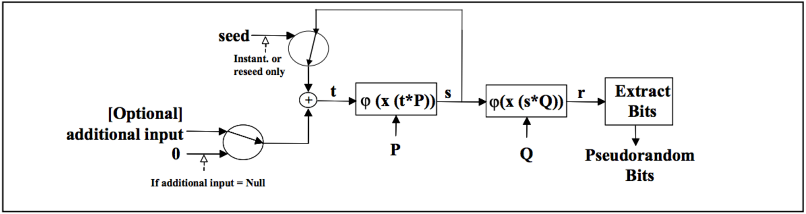
- Let ϕ be a function that truncates $x(P)$ to bits

Dual EC


seed
Instant. or
reseed only
[Optional]
additional input
0
If additional input = Null
t
φ (x (t*P))
s
φ(x (s*Q))
r
Extract
Bits
P
Q
Pseudorandom
Bits

Dual EC

A Security Analysis of the NIST SP 800-90 Elliptic Curve Random Number Generator

Daniel R. L. Brown* and Kristian Gjøsteen†

February 15, 2007

Figure: https://eprint.iacr.org/2007/048.pdf

This is provably biased if you know DLOG $\log_P Q$

DUAL_EC Backdoor (Simplified)

The user

- Two parameters
  (P, Q)
- Compute next state
  $s_{i+1} = P^{s_i} \ mod \ N$
- Compute next output
  $r_i = Q^{s_i} \ mod \ N$

The attacker

- Keep x such that
  $P = Q^x \ mod \ N$
- Observe any output
  r_i
- Compute next state
  $s_{i+1} = r_i^x \ mod \ N$
- Predict all future outputs!

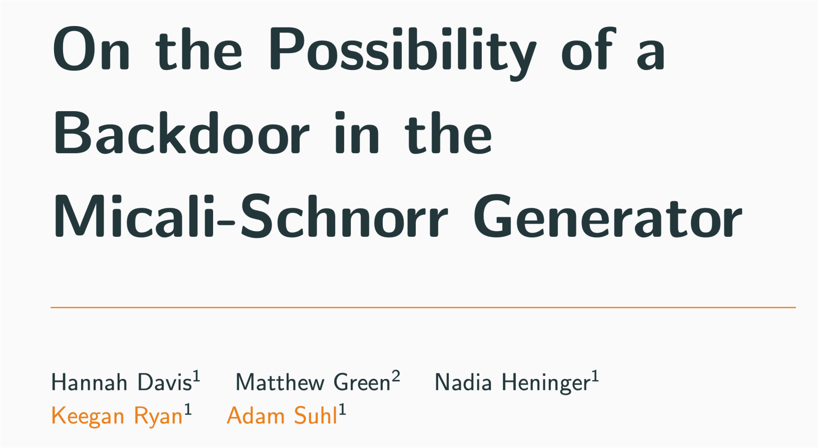
$$s_{i+1} = P^{s_i} = (Q^x)^{s_i} = (Q^{s_i})^x = {r_i}^x \text{ mod N}$$

Figure: `https://www.cs.au.dk/~orlandi/orlandi_backdoors.pdf`

Matthew Green's Blog

- Dual-EC-DRBG: https://blog.cryptographyengineering.com/2013/09/18/the-many-flaws-of-dualecdrbg
- RSA warning: https://blog.cryptographyengineering.com/2013/09/20/rsa-warns-developers-against-its-own
- NSA random number: https://blog.cryptographyengineering.com/2013/12/28/a-few-more-notes-on-nsa-random-number
- Juniper backdoor: https://blog.cryptographyengineering.com/2015/12/22/on-juniper-backdoor

Micali-Schnorr?

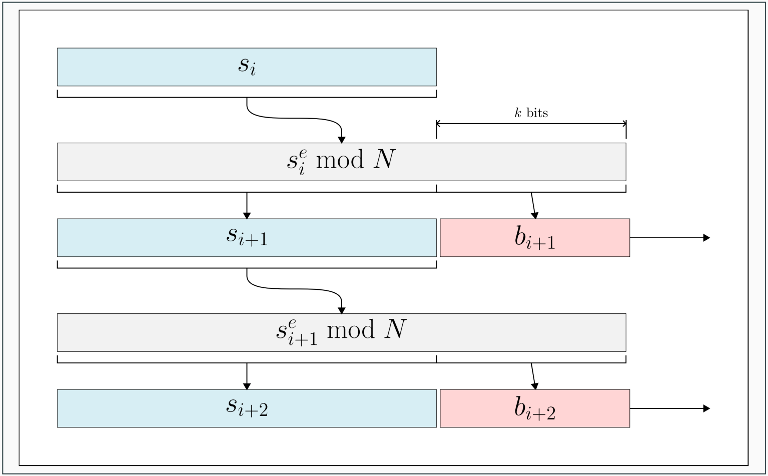


On the Possibility of a Backdoor in the Micali-Schnorr Generator

Hannah Davis[1] Matthew Green[2] Nadia Heninger[1]
Keegan Ryan[1] Adam Suhl[1]

Figure: paper: https://eprint.iacr.org/2023/440.pdf, talk: https://www.youtube.com/watch?v=608NQdTn39Q&t=2629s, slides: https://iacr.org/submit/files/slides/2023/rwc/rwc2023/119/slides.pdf

Micali-Schnorr?



Unclear how to recover the state using RSA decryption.

Questions?